Validation of Bubble Distribution Measurements of the ABS Acoustic Bubble Spectrometer® with High Speed Video Photography

G. L. Chahine, K. M. Kalumuck, J-Y Cheng and G. S. Frederick
DYNAFLOW, INC.
10621-J Iron Bridge Road, Jessup, MD 20794
email: info@dynaflow-inc.com **http://www.dynaflow-inc.com**

Abstract

Measurement of the bubble size distribution in a liquid is very important for cavitation inception studies. In this paper we describe an acoustics based device, the ABS Acoustic Bubble Spectrometer® that measures bubble size distributions and void fractions in liquids based on the measurement of sound propagation through the tested liquid. Short monochromatic bursts of sound at different frequencies are generated by a transmitting hydrophone and received by a second hydrophone after passage through the liquid. These signals are processed and analyzed to obtain the frequency dependent attenuation and phase velocities of the acoustic waves. From these, the bubble size distribution (number of bubbles versus size) is obtained following solution of an inverse problem. In order to validate a new implementation of the instrument software, a fundamental experiment is conducted. Bubbles are generated in a controlled fashion, and then carefully mixed into a uniform distribution in a flowing system. A high-speed micro-video system is used to take videos of the bubbles at the same time and within the test volume interrogated by the ABS system. Both the acoustic data and the video frames are then analyzed using many datasets under the same conditions, and the results are compared. The two methods are seen to provide very close results within their limits of resolution and within the bubble distribution variations in the liquid. The ABS provides results very close to the time-consuming micro video photography in near real-time in a much more cost-effective fashion.

1 Introduction

Determination of the bubble population in a sample of liquid is important in many fields, the most relevant here being cavitation. Several techniques have been used to-date [1-13] and can be divided into optical studies (photography, holography, scattering techniques, ..etc.), acoustical studies (scattering, attenuation, dispersion, ..etc.), and others including electrical impedance and cavitation susceptibility meters. Acoustical methods are inverse methods, relying on the fact that bubbles have a strong effect on the propagation of acoustic waves. The acoustical cross-section of a bubble is three to four orders of magnitude greater than its geometrical cross-section [2]. Acoustical techniques are relatively simple, and applicable to much larger liquid samples. Additionally, liquids are generally much more transparent to the passage of acoustic waves than they are to light.

The predictions of existing acoustical and optical techniques differ widely [6]; for instance, the acoustical method of Wildt [14] over-predicts the bubble population density by as much as two orders of magnitude at small radii, and under-predicts it significantly at larger radii. The error lies in the procedures used to infer the bubble population from the measurements. The method used here has been shown to be a consistent method for obtaining the bubble population from measurements [15, 18-20].

Using a set of effective equations, derived by taking the limit of the complete equations of motion to small bubble volume fractions [16], a dispersion relation for bubbly fluids was developed by Commander and Prosperetti [17]. This relationship was used to obtain the attenuation and phase velocity for given bubble populations and was compared very favorably with measurements. They also found that the computed attenuation and phase velocity was quite sensitive to the bubble population distribution.

Here, we use the inverse of this procedure and obtain two integral equations for the bubble density in terms of the phase velocity and attenuation. Solution of these equations, using measured values of the attenuation and change in phase velocity, allows computation of the bubble population. The problem faced in the solution of these equations is that the equations are ill-posed. We considered in [15,20] several approaches for solving this ill-posed problem, and found that among the approaches tested one based on constrained minimization worked best.

2 Background of the Method

Consider a bubbly medium consisting of a liquid of sound speed c_l containing spherical bubbles of different radii. The bubble size distribution is characterized by the bubble population density, *N(a)*, such that

$$\int_{a_1}^{a_2} N(a)da = Total\ Number\ of\ bubbles\ per\ unit\ volume. \quad (1)$$

When sound of frequency ϖ propagates through the bubbly medium, the bubbles oscillate and extract and re-radiate energy into the medium, thereby making it dispersive. Each bubble acts as an oscillator with a natural frequency ϖ_0 and a damping constant b that depend upon the imposed frequency ϖ and the bubble radius, a. A dispersion relation relates the complex sound speed, c_m, in the mixture to the sound speed in the liquid, c_l, as follows, with i representing the imaginary unit:

$$\frac{c_l^2}{c_m^2} = 1 + 4\pi \int_{a_1}^{a_2} \frac{aN(a)}{\varpi_0^2(a) - \varpi^2 + 2ib(a)\varpi} da. \tag{2}$$

Replacing in (2) ϖ_0 and b with their expressions obtained in [19] we obtain with $\frac{c_l}{c_m} = u - iv$:

$$\begin{aligned} &\int_a^a k_1(a,\varpi)N(a)da = u^2 - v^2 - 1; \qquad k_1 = \frac{\left(\varpi_0^2 - \varpi^2\right)a}{\left(\varpi_0^2 - \varpi^2\right)^2 + 4b^2\varpi^2}, \\ &\int_a^a k_2(a,\varpi)N(a)da = uv, \qquad k_2 = \frac{b\varpi a}{\left(\varpi_0^2 - \varpi^2\right)^2 + 4b^2\varpi^2}. \end{aligned} \tag{3}$$

The quantities u and v may be obtained by measuring the phase velocity, c_m , and the attenuation, A, of the wave in the bubbly liquid. The attenuation A, in dB per unit length, is given by

$$A = 20\log_{10} e\left(\frac{\varpi v}{c_l}\right). \tag{4}$$

3 The Acoustic Bubble Spectrometer Technique

3.1 *Experimental Method*

The ABS Acoustic Bubble Spectrometer extracts the bubble population from acoustic measurements of the phase velocity, c_m , and the attenuation, A, made at several insonifying frequencies. The device consists of a set of two hydrophones connected to data acquisition and control boards resident on a personal computer. Data board control signal generation by the first hydrophone and signal reception by the second hydrophone. Short monochromatic bursts of sound at different frequencies are generated by the transmitting hydrophone and received by the second hydrophone after passage through the bubbly liquid. These signals are processed and analyzed utilizing specialized copyrighted software algorithms that we have developed to obtain the attenuation and phase velocities of the acoustic waves, and, from these, the bubble size distribution. The PC and its resident signal generation and data acquisition hardware synchronize and control the measurements as well as perform the data analysis. A sketch of the procedure is shown in Figure 1.

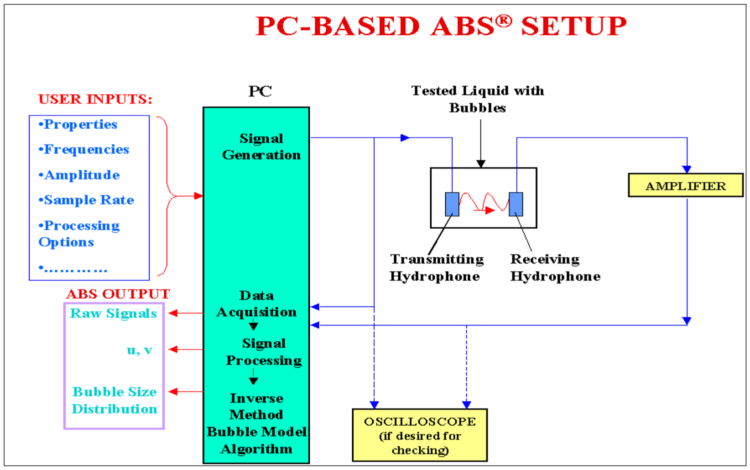


Figure 1. Sketch of the ABS Acoustic Bubble Spectrometer® Method

Measurements are conducted with the aid of a Graphical User Interface, where all physical, experimental, and analytical parameters are input by the user via a series of dialog boxes. Both raw and processed data from experiments can be saved to disk for future use. The results are displayed graphically by the interface in real time and can also be stored or printed.

3.2 *Inverse Problem Solution*

Once a series of measurements of sound speed and attenuation in the bubbly medium is obtained at a set of frequencies covering the range of interest, the inverse problem consists of the determination of the bubble size distribution, corresponding to these measurements. Such an inverse problem is difficult to address and is usually ill-posed, that is small variations in the measured quantities may result in large variations in the sought distribution. Since experiments are prone to measurement errors and numerical computations are subject to round off and other errors, this may result in the solution oscillating wildly when refining discretization until finally the "solution" has little relation to the original data. It is thus necessary to regularize the problem. We have solved this issue using constrained optimization methods [15,18-20]

4 Experiments

4.1 *Experimental Setup*

Over the last few years we have conducted many experiments to validate and improve the accuracy of the ABS Acoustic Bubble Spectrometer [23-28]. These experiments have consisted mainly of using electrolysis bubbles and bubbles injected by various means in an otherwise quiescent liquid [23-25], or bubbles generated by the impact and penetration of a water jet on a free surface, or air injection underwater [26,28].

The major difficulty in any validation test is to have the measurements with the technique being validated and those with the validating techniques taken for the same interrogated volume of the mixture and at the same instant. This is in fact almost impossible. One then tries to minimize the differences between the space and time constraints of both techniques. Here, the difficulty is further compounded by the variations in time and space of the bubble size distribution in the liquid. Since microphotography is still the most accurate and preferred technique for validation, there is a major compromise between the desire to measure the smallest bubble sizes possible and to sample a large enough volume of the liquid to ensure that the micro-photographic measurement is representative of the much larger volume interrogated by the relatively more global ABS technique.

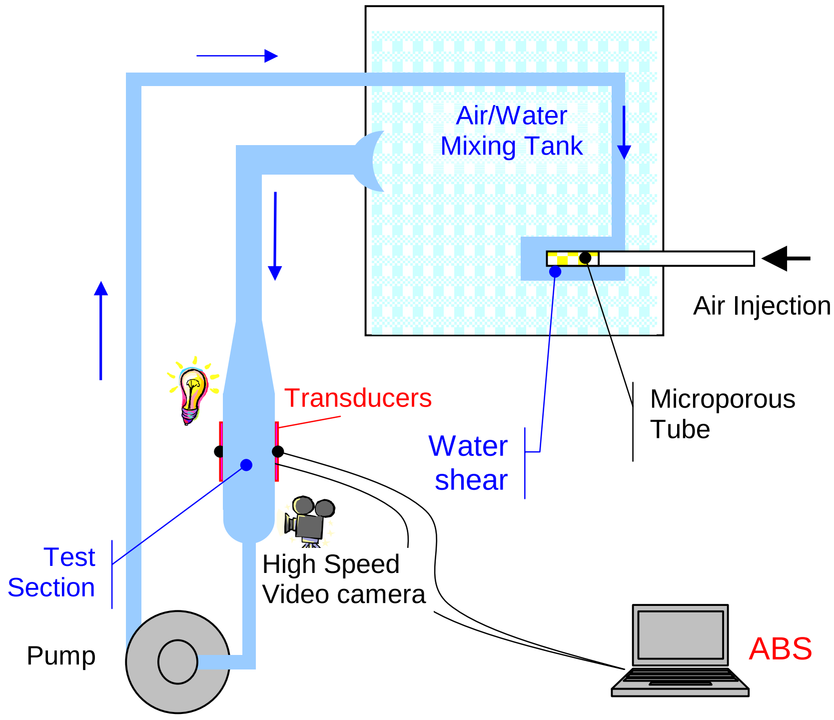


Figure 2. Sketch of the experimental setup for the ABS Acoustic Bubble Spectrometer validation studies

The experimental set-up used in the present study was designed to minimize the above-described difficulties, and to enable validation of the ABS in the case of a flowing bubbly liquid. Figure 2 shows a sketch of the set-up, while Figures 3 and 4 show pictures of the set-up and the bubble injector.

An *8 mm* diameter micro-porous tube, DYNAPERM®, with pore sizes *1* to *10* μm, is used for air injection into a $35\times56\times65$ cm^3 Plexiglas mixing chamber. The tube is placed in a *16 mm* Plexiglas pipe where water is injected at various speeds and used to shear off the bubbles ejecting from the micropores of the DYNAPERM® tube in order to generate smaller size bubbles [28]. The bubble cloud coming out of the water/air injector fills out the mixing tank, and produces a bubble distribution that is a function of the geometrical configuration of the tank, bubble generator, and free surface. Any large bubbles surviving the shearing action or resulting from subsequent bubble coalescence, rise to the free surface of the tank and are thus eliminated from the subsequent analysis.

In one side of the mixing tank, and in this case close to the tank bottom to avoid any large recirculating bubbles, the bubbly mixture is sucked through a 2 *cm* pipe into a $7.5\times7.5\times25$ cm^3 Plexiglas test section. The mixture is then guided through a *3-degree* angle gentle diffuser of length *25 cm* from the 2 *cm* pipe into the test section. The objective is to create a smooth transition into a laminar parallel flow in the test section, in order to minimize as much as possible unsteadiness of the bubble distribution in the flow. Downstream of the test section a pump takes the liquid from the suction port through the test section and back into the mixing chamber through the water injection tube surrounding the DYNAPERM® tube and used to shear the bubbles from the micropores.

Two hydrophones in the form of flat transducers made of piezoelectric composite materials embedded in polyurethane for waterproofing and shape-forming are mounted in two sides of the test section, and are insulated acoustically from the structure by cork layers. One of the transducers acts as the emitter side of the ABS system while the other one acts as the receiver. The size of the active section of each hydrophone is 5×5 cm^2.

Figure 4. Photograph of the test section and the upstream diffuser. In this case the flow was directed downward.

Figure 5. Photograph of the bubble injector: close up view of a) DYNAPERM® tube (white) and water shear injection, and b) injector in operation.

The other two sides of the Plexiglas test section are used for optical observation and measurement in the same measuring volume as the ABS. Microphotography is used to enable counting the bubbles optically. A Redlake high-speed video camera (typically operated at 1,000 frames per second) with a macroscopic lens is used to take a series of pictures of the bubble distribution as a function of time. These images are then analyzed semi-automatically using the data analysis system of the video-camera. The procedure is to analyze one frame, select in that frame a characteristic bubble at the upstream side of the visualized area, skip all following frames in which the same bubble is visible, (i.e. until the bubble has left the field of view,) then select the following frame to analyze. Usually, about 20 such frames were analyzed to provide one data point; all bubbles in focus are measured and counted to generate a bar chart of the bubble size distribution. The bubbles are then grouped into bins that corresponded to those obtained with the ABS analysis in order to enable a side-by-side comparison. The video image is a quasi-2D representation of the real 3D bubbly flow field. However, it has volume information through the depth of the optical field considered; that is objects will appear in focus within a certain depth, $\pm\delta$, of the focus plane. δ is measured initially in the same facility using a thin wire. In order to compute the number of bubbles of a given size in a unit volume, we use the fact that the counted bubbles were observed in a volume of size δwh, where w and h are the width and height of the measurement area (usually of the order of *0.5cm* each).

The ABS measurements were done using the in-wall embedded hydrophones. A series of signal bursts (typically 5-10 periods each) at various frequencies (typically 20 pre-selected frequencies) within the capabilities of the hydrophones (typically between 10 KHz and 250kHz) and the data acquisition board (here 1.1MHz) are used[1] to excite the first hydrophone and are received by the second hydrophone. The ABS software then computes the sound phase speed and the attenuation, and deduces the bubble size distribution. As allowed by the software the experiments were conducted using either a single series of frequencies, or an average over as many as 100 series. In this case, the software considers the resulting values of u and v for each series, (see Figure 5 for

[1] *In the ABS, the user defines in a pull down menu the number of pulses to emit for each frequency, the number of frequencies, and selects the frequencies to be used to drive the experiment.*

a screen shot of these quantities) then generates average curves *u(f)* and *v(f)*, and deduces from this the average bubble population. Obviously, using such a large number of repetitions is not too meaningful if the bubble population is very unsteady. The measured bubble sizes and numbers are assumed to be uniformly distributed in the volume between the two transducers, that is here, in the volume $5 \times 5 \times 7.5\ cm^3$. This volume is used to deduce the bubble size distribution in a unit volume of the tested liquid.

From the above description, the reader should realize that despite all efforts there still are two inherent discrepancies between the two methods. One is the difference between the measurement volumes, about 190 cm^3 for the ABS and about $50\ mm^3$ for the microphotography. The second difference can be the duration of the measurement, which is about a second for the ABS (could be as short as *100 ms*) and a few seconds for the video photography.

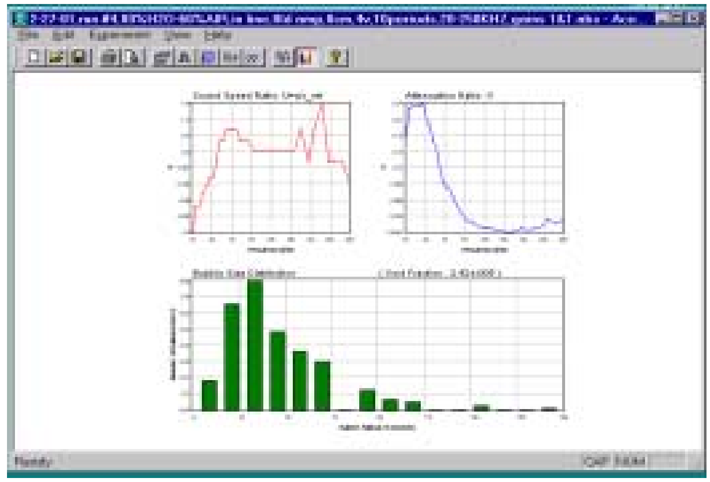


Figure 6. Screen shot of the ABS graphical user interface showing, for one of the case studies reported here, the sound speed ratio, u, and the attenuation ratio, v, defined above in Equation (3). Also shown is the resulting bubble size distribution after solution of the inverse problem by the ABS software.

4.2 Experimental Results

A series of tests was run using various combinations of airflow rates into the DYNAPERM® tube and water flow rates through the pump. This combination allows for various levels of shear and for different void fractions. Since this paper mainly aims at showing calibration comparisons between the ABS results and microphotography, we will not get into all the details of the study, but show some illustrative examples. Some of the figures shown below have information on the flow rates in percent of the maximum flow rates. To make this clearer, for most of the cases shown the configuration of the injection tubes is such that 100% water flow corresponds to an average shear velocity in the water flow injection tube of *3 m/s*. A 100% air flow corresponds to an injection average speed based on the total surface area of the DYNAPERM® tube of *0.05 m/s*. Figure 4 shows a typical example of the data obtained when the measurements are done several times under the same conditions within a few minutes apart, and where to the naked eye the water / bubble mixture appearance looked quite uniform and did not apparently change with time. It is quite obvious that on the local level, the details of the bubble size distribution spectra (number of bubbles in a unit volume of a given size) despite appearances change significantly between one measurement and the next. However, the overall trend does not change much.

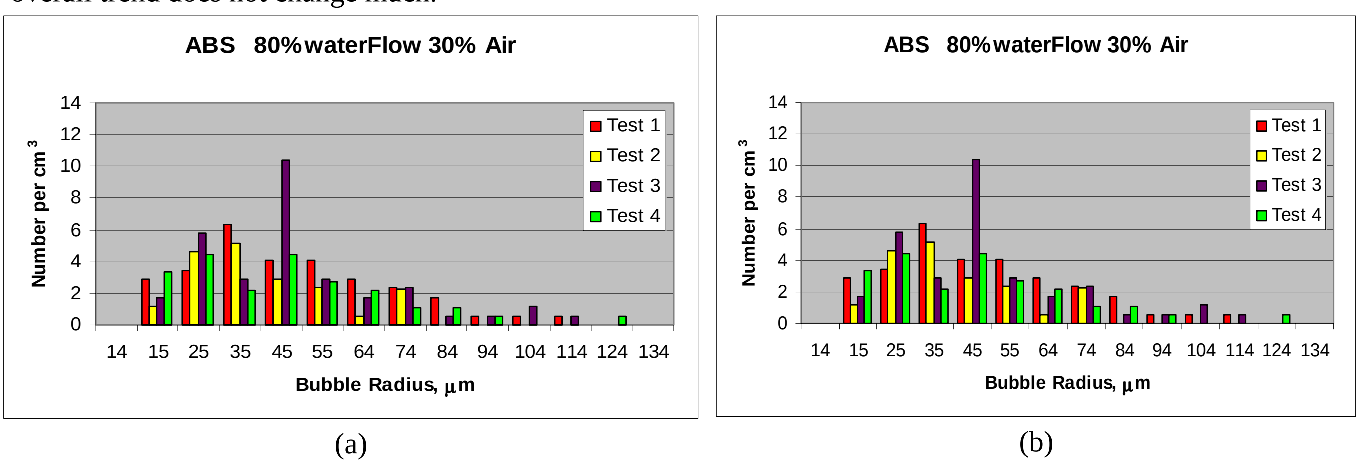


Figure 6. Illustration of the degree of variation in the bubble size distribution in the measurement area using a) one set of measurement each time with the ABS Acoustic Bubble Spectrometer, and b) the micro video photography where each measurement is obtained using 55 frames.

Figure 7 shows a comparison between the ABS and the photography results when an average is made over six ABS computations and six video photo analyses of 22 images each. (In figure 6 we have only shown four of these measurements to avoid more clutter in the bar chart). Here, we can see that the two methods give very close results, even though no attempt was made to adjust any of the parameters such as the value used for the depth of field or the intensity level selected to decide if an image was in focus or not. The decision on whether a bubble was in focus or not was done visually by the same operator independently of any knowledge of the ABS results. There definitely is room for improvement in that area but this was not the objective of the study here. However, the ABS appears to capture extremely well the characteristic bubble sizes. The number of bubbles of a given size appears also to be in general quite close to the optical observations. Discrepancies, however, exist but are definitely within the margin of variation in the results of each of the observation method. These discrepancies appear to be related to actual variations in the bubble population and not so much due to errors in either of the measurement methods.

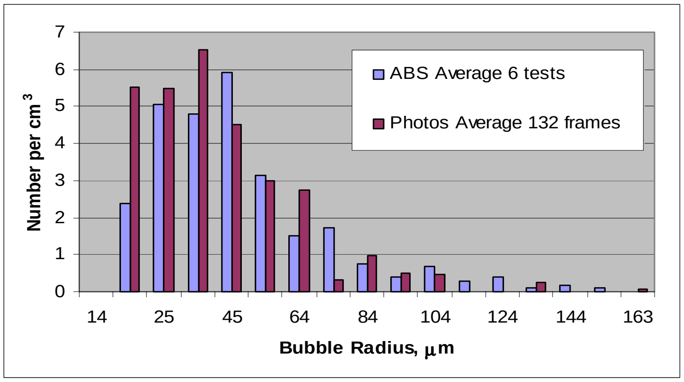
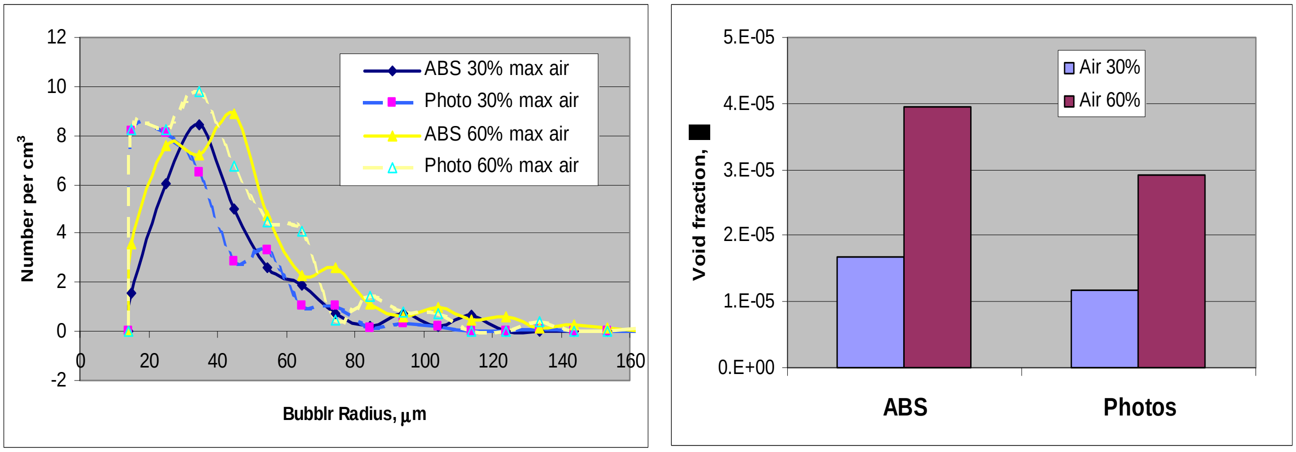


Figure 7. Comparison between ABS measurements and micro video photography averaging six repetitions.

Figure 8 shows the effect of doubling the amount of air injected into the mixing chamber. As measured directly with the video photography and indirectly with the Acoustic Bubble Spectrometer, this results in a modification of the shape of the bubble size distribution. Instead of a simple peak in the distribution, we observe that two peaks are formed on each side of the previous one. This is a result of the complex interaction between the injected air stream from the micropores and the shearing action of the surrounding water flow. It is quite encouraging that both methods capture the same trend and again appear to give very close results within the variations of the experimental bubble/liquid distribution.

Both the ABS bubble size distribution measurements and the micro video photography measurements were also used to compute the void fraction that corresponds to each measurement. This is simply computed by summing up the volume of all the bubbles detected or seen in a unit liquid volume. Figure 8b shows that as expected the computed void fraction measured by both methods approximately doubles as the air injection flow rate doubles. However, one unexpected result is that the computation of the void fraction in that fashion is strongly prone to errors. In fact, the void fraction calculations are very sensitive to the largest bubbles, to which (due to their very low number) most probably the assumed statistics do not apply, e.g. observing one large *150µm* bubble in a sampling volume of *10* cm^3 does not necessarily mean that we have *100* such bubble in *1 liter*. Because of this sensitivity the differences between the ABS and the photographic method are accentuated in the calculation of α.



(a) (b)

Figure 8. Sensitivity of a) the bubble size distribution measurements and b) the resulting void fraction, to a doubling in the flow rate of air through the DYNAPERM® *microporous tube.*

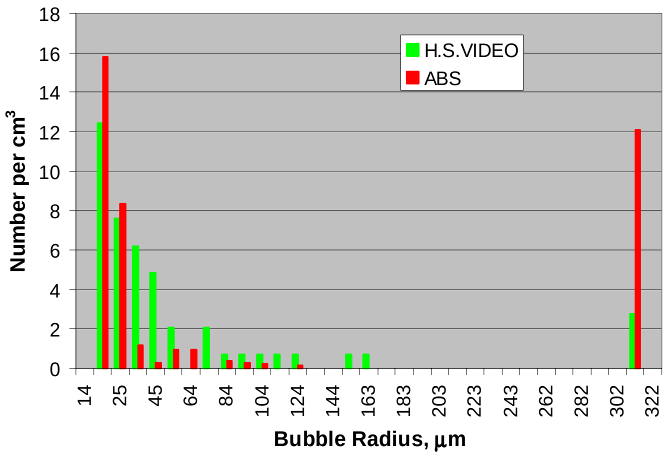


Figure 9. Bubble size distribution in a condition where large bubbles were injected into the test section. Comparison between ABS results and video photography analysis.

In order to test the system when there are two very different peaks in the bubble size distribution, the position of the air generator was on purpose misplaced in order to force some large bubbles to be directly injected in the suction end upstream of the test section. In this case, the configuration shown in Figures 2 and 3 resulted in bubbles accumulating in and above the test section due to gravitational forces on the larger bubbles, which opposed the downward motion of these bubbles. In order to still be able to conduct the measurement with a uniform flow, the test section was turned upside down to have the feed section to the diffuser come from below and the test section on the top. In this case, gravity force was in the same direction as the direction of the flow and the problem of bubble accumulation was eliminated. Figure 9, shows the resulting ABS determined bubble sizes and the direct optical measurements. Again we can see a very good correspondence between the two methods in the detected bubble sizes, with discrepancies in the bubble number in the same range as the repetition error in the experimental realization of the same bubble / liquid configuration.

5 Conclusions

Since bubble size distribution can play an important factor in cavitation studies especially in cavitation inception. It is very useful to be able to characterize the studied liquid for its properties in terms of not only overall air content or dissolved oxygen, but also in terms of the actual detail of the bubble size distribution. The instrument tested in this study, the ABS Acoustic Bubble Spectrometer®, appears to have important advantages that make it useful. It allows near real time on line measurement of the bubble size distribution. It can be designed to be non-intrusive, such as here, where the transducers were located in the walls of the test section. It has the advantage of examining a large volume of the liquid if necessary. More important, the study presented here shows that the system, previously tested successfully using synthetic data and in complex configurations, provides very satisfactory results under controlled experiments when compared with a direct simple but time consuming method such as microphotography. Both ways of measuring the bubble size distribution give very close results in terms of bubble sizes, and differences, which are in the range of the scatter errors in the experiment itself, for the bubble numbers. One additional advantage of the method is its flexibility and adaptability to improvements in the hardware. Limitations now are due to the limitations in the frequency responses of the hydrophones and in the data acquisition rates of the PC Cards. Improvement in the existing capabilities of the industry can be rapidly implemented in the modular system.

6 Acknowledgments

The ABS Acoustic Bubble Spectrometer® was initially developed under SBIR Phase I and Phase II grants from the National Science Foundation. We are most grateful to that support. Many people at DYNAFLOW contributed in several ways to further developments.

7 References

1. H. Medwin, "*In situ acoustic measurements of bubble populations in coastal ocean waters,*" J. Geophys. Res. 75, 599-611, 1970.
2. H. Medwin, "*Counting bubbles acoustically: a review,*" Ultrasonics, 15, 7-13, 1977.
3. H. Medwin, "*Acoustical Determination of Bubble-Size Spectra*," J. Acoust. Soc. Am., 62, 1041-1044, 1977.
4. N. Breitz and H. Medwin, "*Instrumentation for in situ acoustical measurements of bubble spectra under breaking waves,*" J. Acoust. Soc. Am., 86, 739-743, 1989.

5. H. Medwin and N.D. Breitz, "*Ambient and transient bubble spectral densities in quiescent seas and under spilling breakers,*" J. Geophys. Res., 94, 12751-12759, 1989.
6. F. MacIntyre, "*On reconciling optical and acoustical bubble spectra in the mixed layer*," in *Oceanic Whitecaps,* edited by E.C. Monahan and G. Macniocaill, Reidell, New York, 75-94, 1986.
7. D.M. Oldenziel, "*A new instrument in cavitation research: the cavitation susceptibility meter*," J. Fluids Engg., 104, 136-142, 1982.
8. S. Vagle and D.M. Farmer, "The measurement of Bubble-Size Distributions by Acoustical Backscatter," J. Atmos. Ocean. Tech., 9, 630-644, 1992.
9. T. Ohern, J. Torczynski, S. Tassin, S.Ceccio, G. Chahine, R. Duraiswami, and K. Sarkar, "*Development of an Electrical Impedance Tomography System for an Air-Water Vertical Bubble Column,*" Proceedings, Forum on Measurement Techniques in Multiphase Flows, ASME IMEC&E, 1995.
10. C.-T. Hsiao, G. Chahine & N. Gumerov, "*An efficient electrical impedance tomography software combining boundary element method and genetic algorithm,*" OptiCON'99 Optimization Software, Methods, and Applications Conference Proceedings, Newport Beach, California, USA, October 14th -15th, 1999.
11. N.A. Gumerov, G.L. Chahine & A.G. Goumilevski, "*Dipole approximation method and its coupling with the regular boundary element method for efficient electrical impedance tomography,*" Boundary Element Technology XIII, C.S. Chen, C.A. Brebbia, and D.W. Pepper (eds.), pp. 217-226, WIT Press, Southampton, UK, 1999.
12. G.L. Chahine and Y. Shen, *"Bubble Dynamics and Cavitation Inception in Cavitation Susceptibility Meters,"* Journal of Fluids Engineering, Vol. 108, No 4, pp. 444-453, December 1986.
13. L. d'Agostino, and A.J. Acosta, "On the design of cavitation susceptibility meters," 20th American Towing Tank Conference, 1983.
14. R. Wildt, editor, "*Physics of Sound in the Sea, Part IV,*" National Research Council, 1949.
15. R. Duraiswami, S. Prabhukumar & G.L. Chahine, *"Bubble counting using an inverse acoustic scattering method,"* J. Acoust. Soc. Am., 104, 2699-2717, 1998.
16. R.E. Caflisch, M.J. Miksis, G.C. Papanicolau, and L. Ting, "*Effective Equations for wave propagation in bubbly liquids,*" J. Fluid Mech, 153, 259-273, 1985.
17. K.W. Commander and A. Prosperetti, "*Linear pressure waves in bubbly liquids: Comparison between theory and experiments,*" J. Acoust. Soc. Am., 85, 732-746 1989.
18. R. Duraiswami and G.L. Chahine, "*Bubble density measurement using an inverse acoustic scattering technique,*" NSF SBIR Phase I project report, also Dynaflow Technical Report 92004-1, 1992.
19. S. Prabhukumar, R. Duraiswami, & G.L. Chahine, "*Bubble size measurement using inverse acoustic scattering: Theory & Experiments*," ASME Cavitation & Multiphase Flow Forum, 1996.
20. R. Duraiswami, S. Pabhukumar & G.L. Chahine, "*Development of an Acoustic Bubble Spectrometer (ABS) Using an Acoustic Scattering Technique*," DYNAFLOW, INC. Technical Report 94001-1, July 1996.
21. A. Prosperetti, "*Physics of Acoustic Cavitation,*" in "*Frontiers in Physical Acoustics*", XCIII Corso, Soc. Italiana di Fisica, Bologna, Italy, 145-188, 1986.
22. S.W. Provencher and R.H. Vogel, "*Regularization techniques for inverse problems in molecular biology," in "Numerical treatment of inverse problems in differential and integral equations,*" edited by P. Deuflhard and E. Hairer, Birkhauser, Boston, 1983.
23. C. Ait Hocine and M. Ouarem, "*Bubble Size Measurement Study,*" DYNAFLOW, INC. Internal Technical Report 6.002-31, Oct. 1996.
24. Francois and Maxime, *"Study for Bubble Size Measurement Techniques,"* DYNAFLOW, INC. Internal Technical Report 6.002-46, Feb. 1997.
25. G. L. Chahine, R. Duraiswami, and G. Frederick, "*Detection of Air Bubbles in HP Ink Cartridges Using Dynaflow's Acoustic Bubble Spectrometer Technology,*" DYNAFLOW, INC. Technical Report 97014-1, 1998.
26. O. Poulain and L.-X. Renaux, *"Acoustical and Optical Study of Air Entertainment Due to Jet Impact on a Free Surface,"* DYNAFLOW, INC. Internal Technical Report 6.002-50, Sept. 1998.
27. G.L. Chahine, K.M. Kalumuck, *"Feasibility of Measuring Aphron Microbubbles in Drilling Fluids with Dynaflow's ABS,"* DYNAFLOW, INC. Technical Report 99012-1, 1999.
28. T. Jürgensen and Leonard Gabrielle, "*Study of the parameters influencing the generation of microscopic bubbles", *DYNAFLOW, INC. Internal Technical Report 6.002-50, Oct. 2000.