ON IMPLICIT SUBGRID SCALE MODELING FOR TURBULENT FLOWS

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Large Eddy Simulation (LES) is an effective intermediate approach between DNS and RANS, capable of simulating flow features which cannot be handled with RANS such as significant flow unsteadiness and strong vortex-acoustic couplings, and providing higher accuracy than RANS at reasonable cost but still typically an order of magnitude more expensive. Desirable modeling choices involve selecting an appropriate discretization of the flow problem at hand, such that the LES cut-off lies within the inertial sub-range, and ensuring that a smooth transition can be enforced at the cut-off. The main assumptions of LES are: (i) that transport is largely governed by large-scale unsteady features and that such dominant features of the flow can be resolved, (ii) that the less-demanding accounting of the small-scale flow features can be undertaken by using suitable Sub Grid Scale (SGS) models. In the absence of an accepted universal theory of turbulence, the development and improvement of SGS models are unavoidably *pragmatic* and based on the rational use of empirical information. Classical approaches have included many proposals ranging from, inherently-limited eddy-viscosity formulations, to more sophisticated and accurate mixed models, e.g., [1].

For the sake of the discussion, we restrict ourselves to the incompressible regime, and focus on the crucial LES closure issue of modeling the SGS stress tensor, $\mathbf{B} = \overline{\mathbf{v}\quad\mathbf{v}} \square \bar{\mathbf{v}}_N \quad \bar{\mathbf{v}}_N$, which needs to be prescribed in terms of the represented-filtered velocity $\bar{\mathbf{v}}_N$, where the upper bar is denotes filtered quantities. The SGS stress tensor can be conveniently recast in the form $\mathbf{B} = (\overline{\mathbf{v}\quad\mathbf{v}} \square \overline{\mathbf{v}_N\quad\mathbf{v}_N}) + (\overline{\mathbf{v}_N\quad\mathbf{v}_N} \square \bar{\mathbf{v}}_N\quad\bar{\mathbf{v}}_N) = \mathbf{B}_1 + \mathbf{B}_2$, where $\mathbf{B}_1$ denotes the interaction between represented and non-represented scales – which is not known *a priori* – and therefore *must* be modeled, whereas $\mathbf{B}_2$ relates to the interaction between filtered and discretized represented scales which can be approximated by prescribing an estimated $\mathbf{v}_N$ in the represented-velocity space (i.e., the solution to the so-called *soft* deconvolution problem), [2]. In this framework, a basic structural SGS model such as the scale-similarity model provides $\mathbf{B}_2$, and the eventual need of mixed models results from the recognition that $\mathbf{B}_2$ is not dissipative enough and a secondary regularization through $\mathbf{B}_1$ is needed, i.e., an approximation to $\mathbf{v}$ in physical-velocity space must be prescribed (the *hard* deconvolution problem).

The main drawback of mixed models relates to their computational complexity, and ultimately, to the fact that *well-resolved* (discretization-independent) LES is prohibitively expensive for the practical flows of interest at moderate-to-high Re. In fact, because of the need to distinctly separate (i.e. resolve) the effects of explicit filtering and SGS reconstruction models from those due to discretization, carrying out such well-resolved LES can typically amount in practice to performing a coarse DNS. As a consequence, it has been argued that the use of hybrid RANS/LES models for realistic whole-domain complex configurations might be unavoidable in the foreseeable future, e.g., [3]. This has recently led many researchers to abandoning the classical LES formulations, shifting the focus directly to the SGS modeling *implicitly* provided by non-linear stabilization achieved algorithmically, through use of a particular class of numerical schemes, or based on regularization of the discretization of the conservation laws, [4].

The traditional approaches motivated by physical considerations on the energy transfer mechanism from resolved to subgrid scales, express $\mathbf{B}_1$ in terms of an appropriate functional $\mathbf{B}_1$ (e.g. an eddy-viscosity SGS model), and seek sufficiently high-order discretization and grid resolution to ensure that their effects are sufficiently small. However, it can also be argued that $\mathbf{B}_1$ might be *implicitly* provided by discretization if non-linear stabilization can be achieved algorithmically, through use of a particular class of numerical schemes, or based on regularization of the discretization of the conservation laws. In fact, most schemes can potentially provide built-in or implicit SGS models enforced by the discretization errors, provided that their leading order terms are dissipative. We are thus lead to the natural question: to what extent can we avoid the (explicit) filtering and modeling phases of LES (i.e., $\mathbf{B}_2\equiv\mathbf{0}$) and focus on the implicit $\mathbf{B}_1$ provided by a suitably-chosen discretization scheme?

Not all implicitly implemented SGS models are expected to work for LES: the numerical scheme has to be constructed such that the leading order truncation errors satisfy physically required SGS-model properties, and hence non-linear discretization procedures will be required. The analogy to be recalled is that of shock-capturing schemes designed under the requirements of convergence to weak solution while satisfying the entropy condition. Finite-volume versions of such schemes can likewise be viewed as relevant for Implicit LES (ILES) if we focus on the small-scale characteristic features of turbulence. In the Monotonically Integrated LES (MILES) approach – see Ref. [5] for a recent survey, the effects of the SGS physics on the resolved scales are incorporated in the functional reconstruction of the convective fluxes using locally-monotonic methods. Analysis based on the modified equations can be used to demonstrate an intriguing feature of MILES, namely that when based on a particular class of flux-limiting schemes, the convection discretization implicitly generates a non-linear tensor-valued eddy-viscosity that acts to stabilize the flow and suppress unphysical oscillations. The MILES performance has been demonstrated in case studies including, 1) canonical flows (forced & decaying homogeneous isotropic turbulence and turbulent channel flows), 2) complex free and wall-constrained flows (mixing layers, jets, wakes, jets and flow past a prolate spheroid), and, 3) extremely complex flows at the frontiers of current unsteady flow simulation capabilities (e.g., submarine hydrodynamics and pollutant dispersion in urban areas).

MILES seeks to emulate the flow features in the high-wave-number end of the inertial subrange region of turbulent flows – characterized by thin filaments of intense vorticity embedded in a background of weak vorticity. We have proposed that emulation of the latter feature be the requirement for implicit SGS models. MILES can thus extend to the more general concept of non-linear ILES in which the functional reconstruction of the convective flux functions is carried out using high-resolution non-linear numerical schemes incorporating a sharp velocity-gradient capturing capability operating at the smallest resolved scales. By focusing on the inviscid inertial-range dynamics and on regularization of the under-resolved flow, ILES thus follows up very naturally on the historical precedent of using this kind of numerical schemes for shock capturing. Challenges for ILES development include developing a common appropriate mathematical and physical framework for its analysis and development, further understanding the connections between implicit SGS model and numerical scheme, and in particular, addressing how to build physics into the numerical scheme to improve on the global ILES performance, i.e., on the implicitly-implemented SGS dissipation & backscatter features. Moreover, additional (explicit) SGS modeling might be needed to address inherently small-scale physical phenomena such as scalar mixing and combustion – which are actually outside the realm of <u>any</u> LES approach: how do we exploit the implicit SGS modeling provided by the numerics, to build efficient "mixed" (explicit/implicit) SGS models ?

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