Subgrid closure for large eddy simulation using one-dimensional turbulence

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1 Abstract

In this talk we describe the development of a new subgrid closure model for large eddy simulation (LES) based on the one-dimensional turbulence model (ODT) of Kerstein [JFM, 2001], and present a comparison of computed results with the experimental data of Comte-Bellot and Corrsin [JFM, 1971] for decaying isotropic turbulence. We also discuss unique advantages of this approach when extended to turbulent reacting flows.

For LES of constant density flows, the subgrid physics to be modeled include energy dissipation, turbulent stresses (both near-wall and bulk flow), backscatter, anisotropy, intermittency, and transition. At present, the "mixed model," which consists of a linear combination of the dynamic Smagorinsky model [Germano *et al.*, Phys. Fluids A, 1991] and the scale similarity model of Bardina *et al.* [AIAA, 1980-1357], might be considered the current state of the art. However, although this approach has been shown to adequately treat energy dissipation and bulk flow stresses, most of the other challenges just mentioned remain problematic. In particular, wall bounded flows have been the Achilles heel of LES owing to anisotropy and the dramatic length scale reduction near the wall. A recent study analyzed in detail the requirements for accurate closure for turbulent channel flow [S. Volker *et al.*, Phys. Fluids, 2002], where it was concluded that it is insufficient solely to capture the transfer of energy from grid resolved to subgrid scales. At a minimum, the closure must also capture the subgrid transport (especially wall normal transport in near-wall flow), subgrid stresses, and subgrid intercomponent transfer due to pressure effects. To the best of our knowledge, no closure approach currently being pursued, except LES/ODT, can demonstrably capture these effects in principle, let alone represent them accurately.

In reacting flows, scalar mixing, thermochemical state history, buoyancy, and multi-component transport effects can be important. An LES must therefore include transport equations for energy and species, which introduces still more subgrid terms that must be closed. Often in practice these terms are handled with the gradient diffusion hypothesis, with diffusivity based on a turbulent Schmidt number. It is important to note, however, that there is nothing which requires the subgrid flux vectors to be aligned with scalar gradient vectors (a key assertion in gradient diffusion). In all cases the subgrid term definitions are decompositions of the filtered non-linear advective term, which is entirely inviscid in nature. It seems desirable, therefore, to seek a model which can address the previously mentioned issues, and enforces a tight coupling of the subgrid scalar field to the subgrid velocity field.

ODT is a novel approach to modeling turbulence that has characteristics uniquely suited to addressing many of the subgrid closure issues associated with LES. As a stand alone model, ODT has found use in several classes of flows, including: compressible shear layers, boundary layers, oceanographic flows, reacting jets, and even astrophysical flows such as Rayleigh-Benard convection in stars. In ODT, the fields defined on its 1d domain evolve by two mechanisms: (1) molecular diffusion, and (2) a sequence of instantaneous transformations, called "eddy events," which represent turbulent stirring. Each eddy event may be interpreted as the model analog of an individual turbulent eddy. In this regard, ODT resembles its predecessor, the linear-eddy model (LEM) [Kerstein, JFM, 1991] for turbulent scalar mixing, making it readily applicable to subgrid scalar transport in reacting flows. In ODT, however, the dependence of the event frequency on

eddy location and length scale evolves dynamically, governed by a probabilistic model, rather than being specified by an assumed frequency distribution.

Recently, Schmidt *et al.* [JCP, 2003] used ODT as the basis for a near-wall LES closure model and tested the approach in turbulent channel flow. This model introduced a system of equations similar to the Prandtl boundary layer equations, with the added feature of a stochastic rearrangement process to simulate wall normal subgrid advection. The more general LES closure proposed here is an outgrowth of ideas developed during that effort, but has important conceptual and procedural differences.

The LES/ODT algorithm can be summarized as follows. A 3d ODT lattice is used as a support for the fully resolved field (a DNS surrogate). The filtered values of the ODT field in each of the three directions are forced to match the LES cell averages through a low wave number adjustment procedure, which does not disturb the high wave number (subgrid) structure. The subgrid stresses are not computed directly. Rather, the time integrated divergence of the stress is modeled with the ODT evolution procedure. This procedure consists of stochastic instantaneous eddy events (which models inviscid subgrid advection), and viscous transport. The difference in the ODT cell average from before and after the evolution procedure provides a measure of the integrated stress divergence (subgrid force). This term shows up on the right hand side of the LES equations. The LES velocity field is corrected for continuity via a projection method; and these corrected velocities become the new targets for the low wave number ODT adjustment, hence closing the cycle.

The new closure method was tested by comparing simulation results against the experimental data of Comte-Bellot and Corrsin ($Re_\tau = 72$) [JFM, 1971] for decaying isotropic turbulence (see figure below). A direct numerical simulation of this experiment is obtainable using a 512^3 grid; therefore, each ODT line contained 512 points. For an N^3 LES, ODT requires $3N^2$ lines (one line for each direction and each column of LES control volumes), or $3N^2 \times 512$ points, for this case.

Overall, satisfactory performance was obtained with the ODT closure model for the Reynolds numbers tested, $Re_\tau = 72$ and 720. [Although not shown here, we have successfully tested this method against the data of Kang *et al.* ($Re_\tau = 720$) [JFM, 2003] using the same model parameters.] Having only tested one canonical flow, a major question facing this method is the constancy of the model parameters (there are two: an eddy rate constant, and a viscous cut-off) for other flows. This issue is not resolved. Despite this question mark, LES/ODT has found success where many synthetic field models have not, namely: correctly predicting energy transfer between the resolved and subgrid scales without the use of an eddy viscosity.

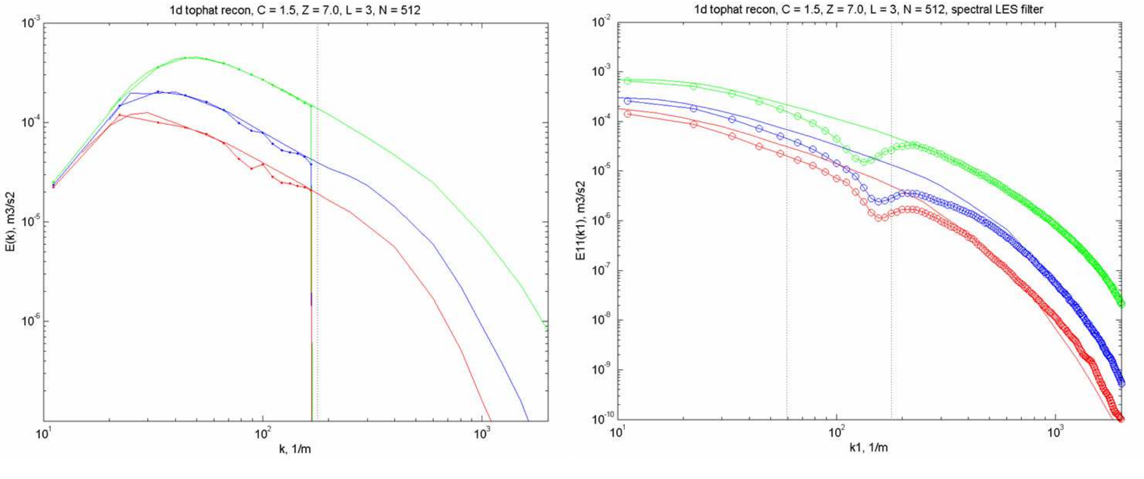

Figure 1: 3d spectra of 32^3 LES (left) and 1d spectra of 512 ODT (right) from a coupled LES/ODT simulation ($3(32)^2 \times 512$ ODT points). The Comte-Bellot and Corrsin data are the solid lines. The simulation results are the dots (left) and circles (right) connected by lines. The "dip" in the ODT spectra is caused by forcing the ODT field to match the LES field up to LES Nyquist limit.