Numerically Lucid Modeling

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What is numerically lucid modeling? It is acknowledging that numerical integration itself involves considerable modeling as a matter of course and accounting for its impact in conducting any additional modeling.

When modeling subgrid effects in a physical system, the effects of numerically integrated the system must be accounted for. In the vast majority of cases, the subgrid modeling is done without regard for the numerical integration. This is only valid under extremely restrictive circumstances. These circumstances are virtually absent from the set of applications that interest the Department of Energy. For problems with discontinuities such as shock waves or material interfaces methods adaptively dissipative must be used.

Fortunately numerical methods used in most applications are built upon our physical understanding rather than simply being numerical approximations. In fact, the best numerical approximations encode a substantial amount of physics directly into the integration procedures. Examples of this are found in upwind and Godunov methods, but the archetypical example is the venerable artificial viscosity.

The question that must be answered is how to characterize the effects of the numerical integration. The technique of modified equation analysis is a means to this end. Even where the numerical solution is (nearly) discontinuous this method is remarkably successful in uncovering the effective subgrid model in a numerical method. In most cases, the numerical integration produces effects that are of the same order as the subgrid effects. Thus numerical integration must be considered hand-in-hand with the modeling.