A COMPUTER STUDY OF ELECTRON-ELECTRON INTERACTION IN HIGH DENSITY ELECTRON BEAMS

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ABSTRACT

High density electron beams are simulated by a computer, and the trajectory displacement and energy broadening caused by electron-electron interaction are investigated computationally. The results are summarized into two empirical formulas which represent dependences of the average trajectory displacement and the average energy broadening on the beam parameters. The results show that the trajectory displacement caused by electron-electron interaction imposes a severe problem on system designers using high density beams, and that energy broadening on the order of leV may well be attributed to electron-electron interaction. The method of simulation is also described.

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1. Introduction

Use of high density electron beams seems to be very promising for fast VLSI pattern exposure, and several groups have already proposed VLSI-oriented pattern exposure schemes using high density beams [1,2, 3,4].

Using high density beams, however, we can not ignore the effect of the Coulomb interaction between beam electrons (so-called electron-electron interaction). In fact, many experiments have revealed that this effect becomes remarkable when the beam current reaches 1μA [5,6, 7]. Since this effect weakens beam focusing and reduces the resolution of the beam, system designers should be aware of such properties if they want to use high density beams.

Because of its importance, many authors investigated the effect of electron-electron interaction theoretically [8,9]. However, it seems, at least for the present author, that the effect is not well clarified yet. One reason is that the phenomenon is a many-body effect and not easy to investigate theoretically. Another reason is that actual beam geometries are complicated, making the analytical calculations difficult. These reasons stimulate us to simulate the phenomenon by computers and study the effect computationally.

This paper describes one such study performed recently. In this work, electron beams very close to actual beams were simulated by a computer, except that the numbers of electrons treated were much smaller than those in actual cases. Since the Coulomb force is long-range, we must take forces between many pairs of electrons into consideration in the simulation. This prevents us from treating more than several thousand electrons at one time by even a fast computer. However, error analysis shows that the accuracy of our results is about 5%.

Electron-electron interaction was investigated computationally by Loeffler and Hudgin [10] about ten years ago. However, their method is considerably different from ours. They first generated unperturbed electron trajectories randomly in which electron-electron interaction was neglected. Then, they estimated the effect of electron-electron interaction by assuming electrons to move along their unperturbed trajectories. In our simulation, motions of electrons are traced by a small time step by taking electron-electron interaction into the equation of motion at each time step. Furthermore, we treat beams having many crossovers, while Loeffler and Hudgin treated beams having only one crossover.

2. Assumptions in simulation

The equation of motion of the i-th electron is

$$m_e \ddot{\vec{r}}_i = \vec{F}_{ext} + \frac{e^2}{4\pi\varepsilon_0} \sum_{j\neq i} \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}, \quad i=1,2,3,\ldots,$$

where m_e is the electron mass, $\vec{r}_j$ is the coordinate vector of the j-th electron, and $\vec{F}_{ext}$ is an external force. The second term in the right side represents electron-electron interaction. We discard the force due to a magnetic field caused by the beam current because we treat only non-relativistic beams. The problem is to solve the above coupled non-linear equations with some initial conditions.

The first assumption we make in our simulation is that electrons are ideally accelerated in such a way that they have a Maxwellian velocity distribution corresponding to the cathode temperature when they are observed in a moving system in which the average electron velocity is zero. This means that we neglect the effect of electron-electron interaction during the acceleration. Although this assumption was adopted in many theoretical calculations, its validity is an open problem. We also assume that the electrons are accelerated until they reach the first crossover and no acceleration field acts on them behind the first crossover.

The second assumption is that the momentum of an electron after the acceleration is much greater than the momentum which the electron receives during its path from the gun to the screen by electron-electron interaction. Since the kinetic energy of an electron after the acceleration is very large in actual beams, this assumption is quite valid. With this assumption, we can treat the electron-electron interaction term as a perturbation. Unperturbed trajectories are those which are determined only by $\vec{F}_{ext}$.

The third assumption is that we can discard interaction between electrons which are very distant from each other. This assumption is necessary because of limited computational power, and its validity is checked computationally.

We simulate beams of circular cross sections which are focused by a uniform coaxial magnetic field with a focal length of 5cm. A uniform focusing magnetic field is very convenient for simulation because we can calculate unperturbed motions of electrons exactly. Furthermore, uniformity of the external force makes the programming quite simple. Note that the motions of electrons near crossovers, where electron-electron interaction is the most effective, are not so dependent on whether the focusing is made by a uniform magnetic field or a combination of electric and magnetic fields. Note further that, in our

scheme, we can simulate nearly divergent beams by only weakening the focusing magnetic field.

Following Loeffler and Hudgin [10], the effect of electron-electron interaction may be divided into three parts, the trajectory displacement $\Delta\vec{r}$, the energy change ΔE, and the angular change $\Delta\alpha$. However, we consider only the trajectory displacement and the energy change because the angular change is not so important in actual system designs.

3. Method of simulation

We characterize an electron beam by the beam current I, the electron acceleration voltage V, the beam radius r_c at the first crossover, the beam semi-angle α at the first crossover, the beam length L, and the cathode temperature T.

Following the assumptions given in a previous section, we emit N electrons randomly at the gun (precisely speaking, at the first crossover) in such a way that they have the beam parameters I, V, r_c, α, and a Maxwellian energy distribution with the temperature T. Therefore, we are not treating beams ranging from the gun to the screen but a group of electrons. The size of the group is only about several millimeters.

These electrons are moved toward the screen discretely by the time step $(L/\bar{v})/n$, where $\bar{v}$ is the average axial velocity of the electrons and n is a large integer. At each time step, the motion of each electron, except for m electrons at each end of the group, is first determined by the focusing magnetic field and then corrected by calculating the forces from the nearest 2m electrons. The correction is performed only for those electrons which have not yet passed through the screen. Therefore, the number of Coulomb force evaluations in one simulation is about $2m(N-2m)n$.

When all electrons reach the screen, we calculate the trajectory displacement $|\Delta\vec{r}|$ at the place of screen and the energy change $|\Delta E|$ of each of the central N-4m electrons in the group by comparing its motion with its unperturbed motion. The 2m electrons at each end of the group are discarded so as to reduce the "end effect." In this way, we get distributions of the trajectory displacements and the energy changes of N-4m electrons.

Let us call the values of the beam parameters "standard" when they are I=2μA, V=20kV, r_c=10μm, α=2mrad., L=30cm, and T=2500°K. By changing each beam parameter with others fixed to the standard values, we simulated many beams under different conditions. In this way, results were obtained for the dependences of the average trajectory displacement $\langle|\Delta\vec{r}|\rangle$ and the average energy change $\langle|\Delta E|\rangle$ on the beam parameters. The range of beam parameters investigated are $0.5\mu A \leqq I \leqq 10\mu A$, $5kV \leqq V \leqq 100kV$, $5\mu m \leqq r_c \leqq 20\mu m$, $5cm \leqq L \leqq 50cm$, $1mrad. \leqq \alpha \leqq 10mrad.$, and $500°K \leqq T \leqq 2500°K$.

The simulation parameters, i.e., N, n, and m, were set in most cases as N=1200, n=1000, and m=50. We checked appropriateness of the values of m and n in beams of length 5cm with other beam parameters fixed to the standard values. We found that, if we changed the value of m from 50 to 100, the value of $<|\Delta\vec{r}|>$ increased by about 2.2% and the value of $<|\Delta E|>$ decreased by about 3.7%. If we changed the value of n from 100 to 200, the values of $<|\Delta\vec{r}|>$ and $<|\Delta E|>$ increased by about 0.4% and 0.6%, respectively. We estimated the amounts of accumulated rounding errors by comparing unperturbed trajectories calculated analytically with those calculated by our simulation program with the time step n=1000. We found that the errors caused by rounding are less than 0.05%. We may conclude from these results that the errors come mainly from smallness of m (2.5∿3.5%) and N ($1/\sqrt{1000}$∿3.2%), hence the accuracy of our results is about 5%.

4. Results of simulation

As we have mentioned above, two distributions are obtained in each simulation: One is of trajectory displacements and the other is of energy changes. These distributions are similar to a Gaussian and an exponential distributions with broadened tails, respectively. For each distribution, we calculate the average and the variance.

Figures 1 to 6 show our results of simulations. In these figures, we used the term "energy broadening" instead of the term "energy change." Throughout these figures, a black dot and a white circle denote average values of the trajectory displacement and the energy change, respectively. Let the average and the variance of a distribution be $\bar{f}$ and v, respectively. In each figure, a vertical line (solid or dashed) represents the range of a distribution: The top and the bottom of such a line represent $\bar{f}+v$ and $\bar{f}-v$, respectively. For ease of readability, a smooth solid curve is fitted to black dots and a smooth dashed curve is fitted to white circles.

We fitted a simple fractional power curve to each parameter dependence. By neglecting the constant terms, the result may be expressed as follows:

$$<|\Delta\vec{r}|> \propto LIV^{-4/3} r_c^{-1/5} \alpha^{-3/4} T^{-1/10}, \quad \text{for } \alpha<5\text{mrad.},$$

$$<|\Delta E|> \propto L^{1/2} I^{1/2} V^{0} r_c^{-1/3} \alpha^{-2/5} T^{0}, \quad \text{for } \alpha<5\text{mrad.}$$

The α dependence tends to be very small for $\alpha>5$mrad.

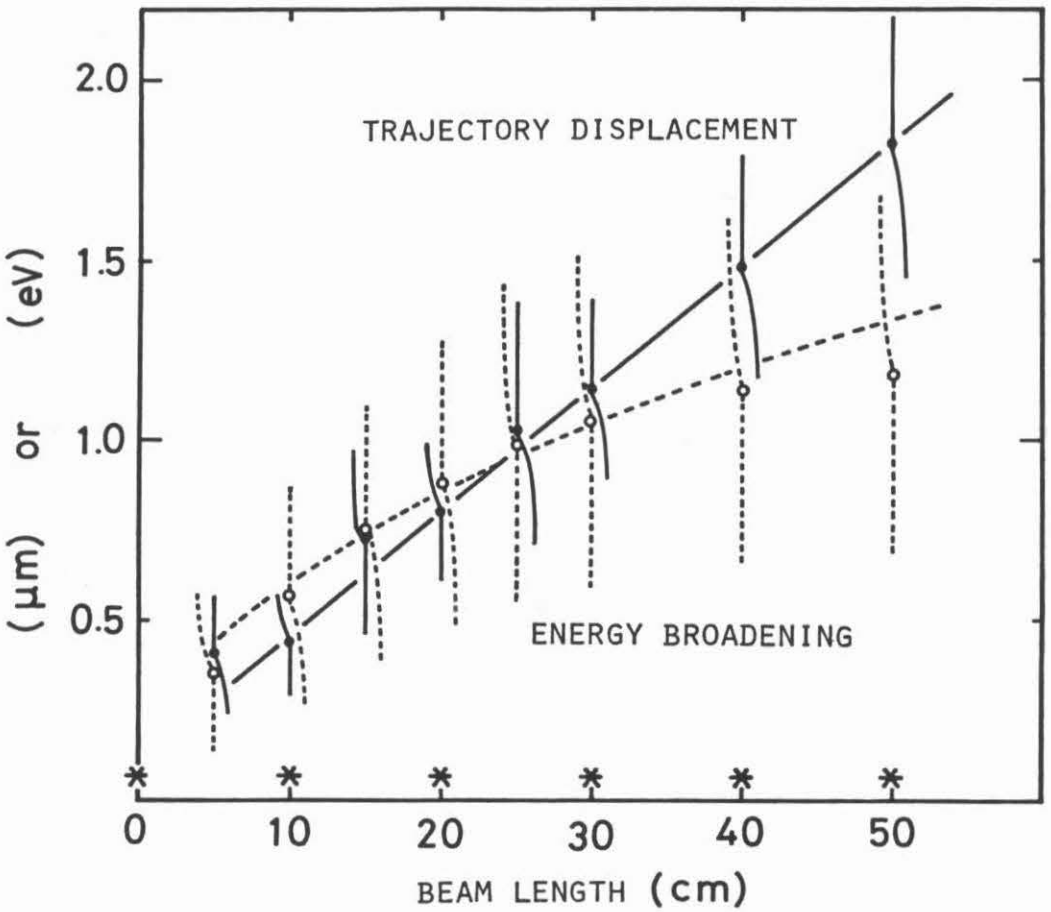


Fig.1 Dependence on the beam length L.
Other beam parameters are I=2μA, V=20kV,
r_C=10μm, α=2mrad., and T=2500°K.
The asterisks denote crossover points.

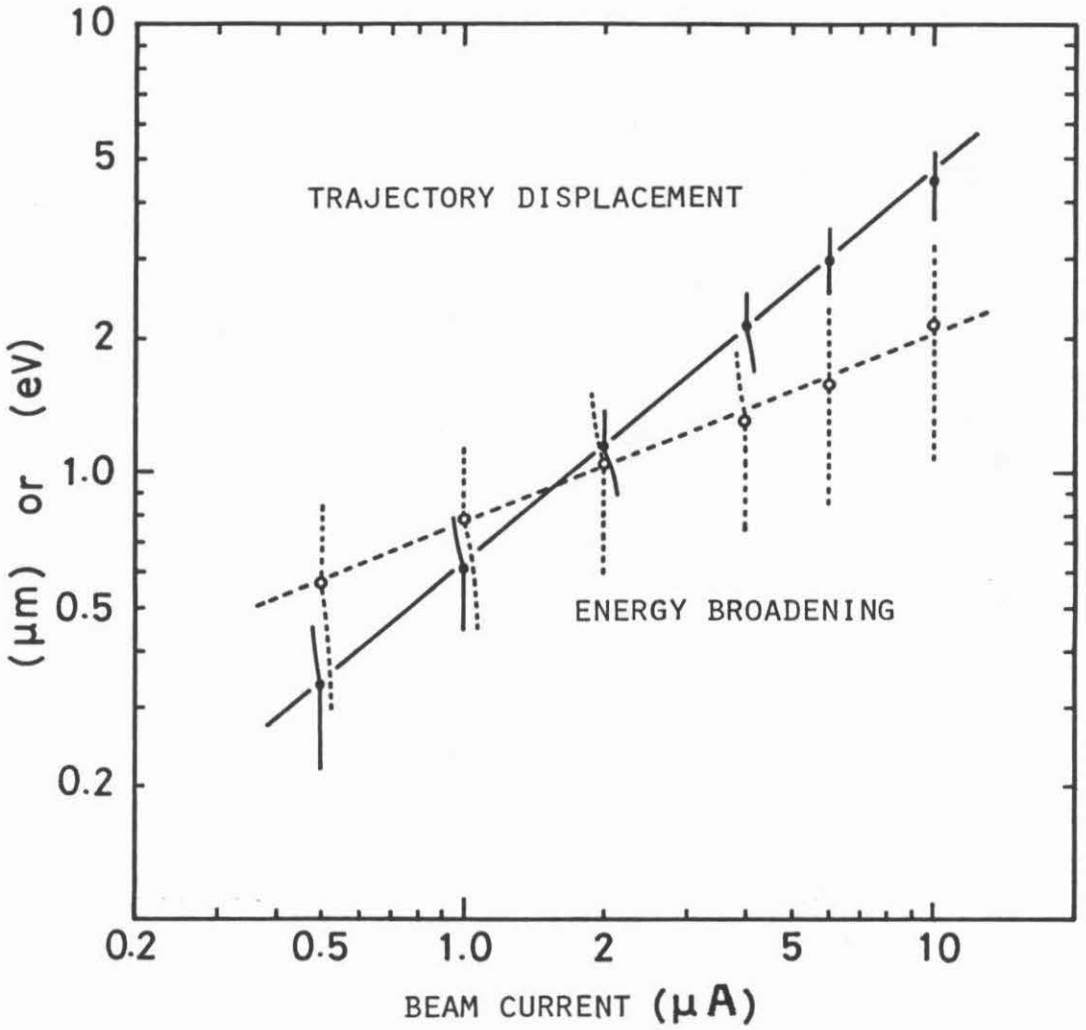


Fig.2 Dependence on the beam current I.
Other beam parameters are V=20kV, r_c=10μm,
α=2mrad., L=30cm, and T=2500°K.

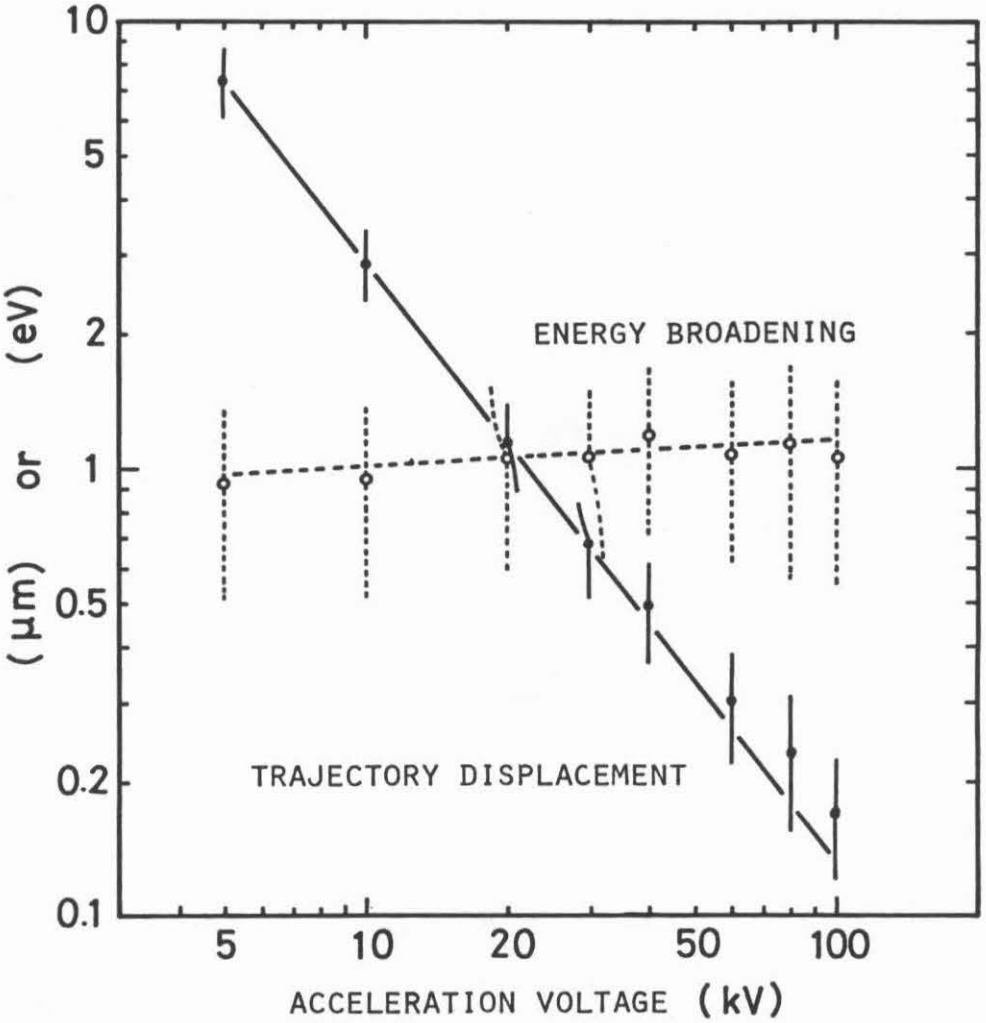


Fig.3 Dependence on the acceleration voltage V. Other beam parameters are I=2μA, r_c=10μm, α=2mrad., L=30cm, and T=2500°K.

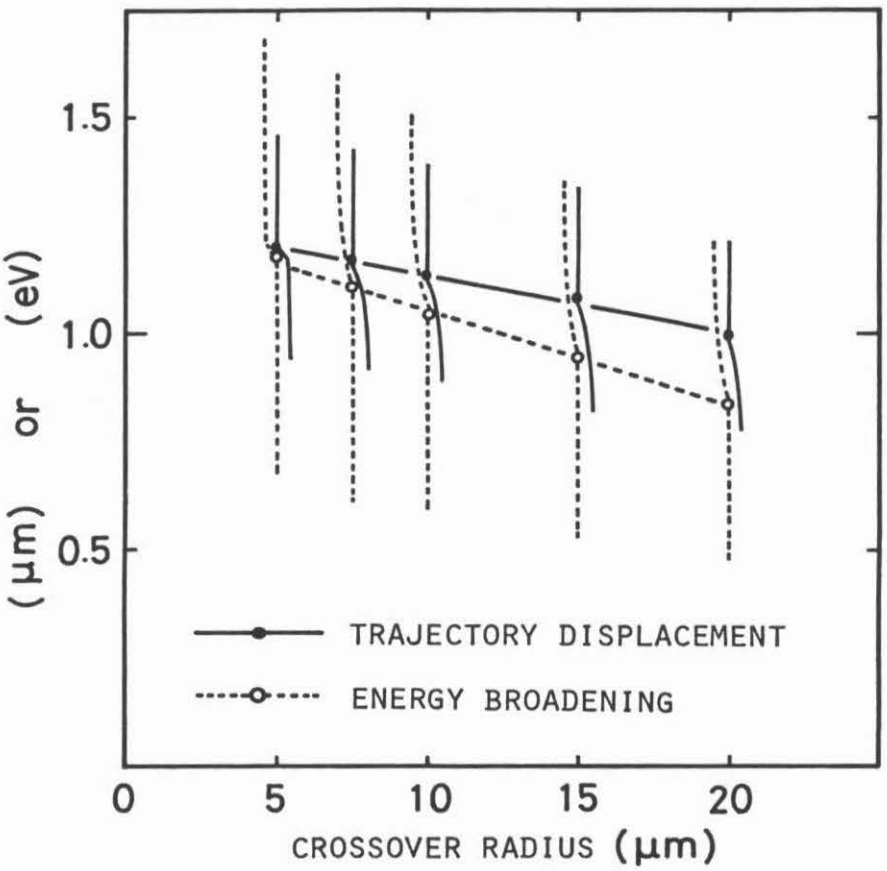
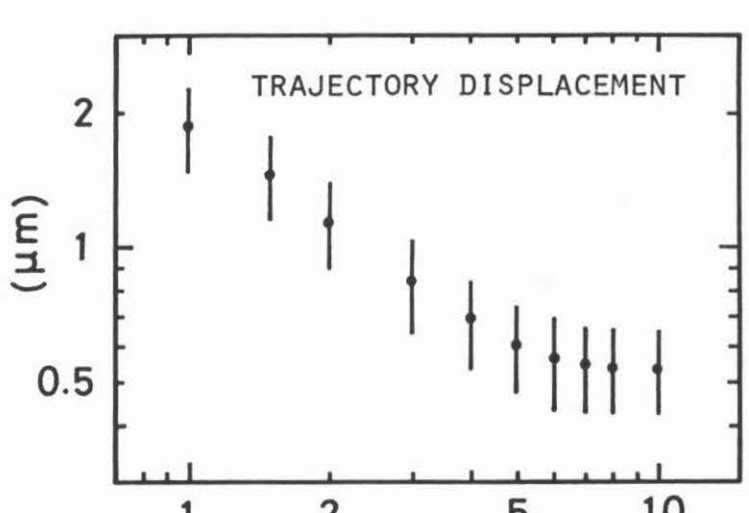


Fig.4 Dependence on the crossover radius r_c.
Other beam parameters are I=2μA, V=20kV,
α=2mrad., L=30cm, and T=2500°K.



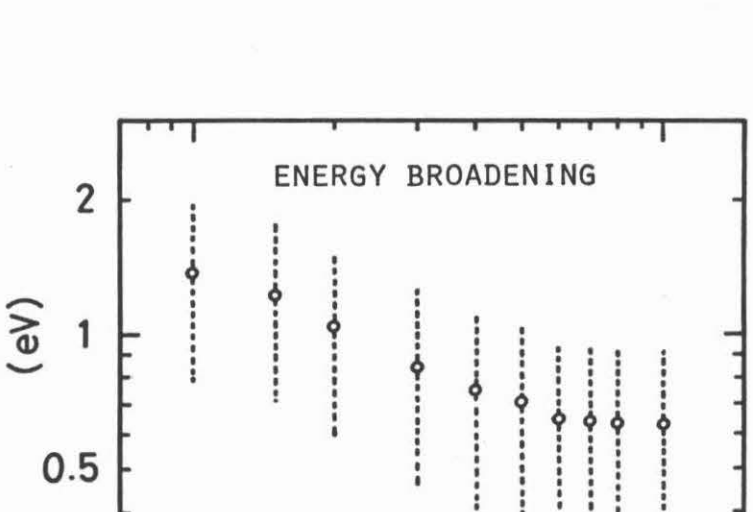


Fig.5 Dependence on the beam semi-angle α. Other beam parameters are I=2μA, V=20kV, r_c=10μm, L=30cm, and T=2500°K.

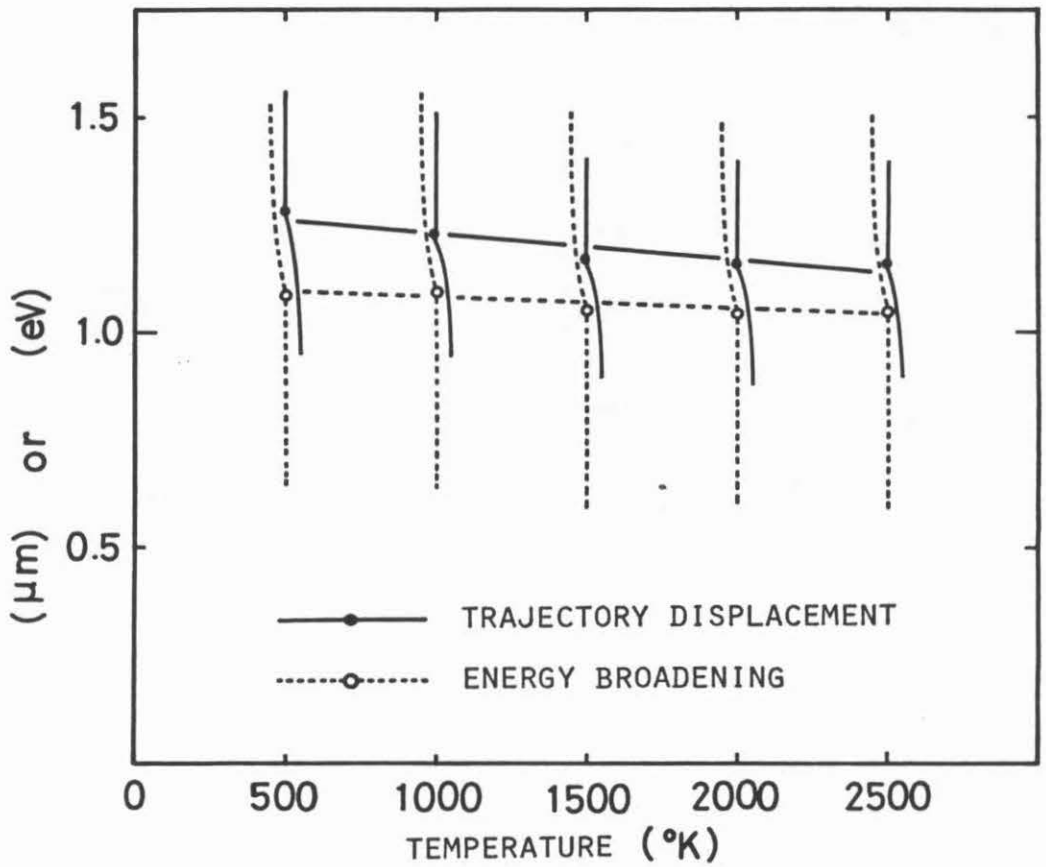


Fig.6. Dependence on the cathode temperature T. Other beam parameters are I=2μA, V=20kV, r_c=10μm, α=2mrad., and L=30cm.

5. Discussions and conclusions

Our results show that, when the beam parameters are "standard," the average trajectory displacement and the average energy broadening are about 1μm and 1eV, respectively. This result is consistent with an experiment [11].

Since electron-electron interaction in electron beams is stochastic, it is very difficult to eliminate the aberration caused by it. However, we can eliminate some parts of trajectory displacements dynamically. The electric field caused by beam electrons has a smoothed part which is produced by an equivalent smeared-out electron distribution. This field causes systematic trajectory displacements, and most parts of which can be eliminated by refocusing. We found that, when the beam parameters were standard, about half of $\langle|\Delta\vec{r}|\rangle$ was eliminated by refocusing.

We compared our calculations with those of Loeffler and Hudgin by simulating nearly divergent beams of length $L \leq 10$cm with other beam parameters fixed to the standard values. We found that our value of $\langle|\Delta\vec{r}|\rangle$ was about two times greater than theirs and our value of $\langle|\Delta E|\rangle$ was about four-fifths of theirs. Considerable parts of these differences are explainable by the difference in beam geometries: Our beams start at the first crossover point while their beam geometries are symmetric with respect to the crossover point. However, several remarkable differences are observable from the dependences on beam parameters. A complete discussion on these differences as well as comparisons with theories will be presented elsewhere.

After Boersch [5] found that the energy distribution of electrons at the screen was broadened, there were many discussions on energy broadening, including critical experiments [12]. Our results, however, show clearly that energy broadening on the order of 1eV may well be attributed to electron-electron interaction.

From the viewpoint of system designers using high density electron beams, trajectory displacement is much more important than energy broadening, as was pointed out by Pfeiffer [13]. In fact, the trajectory displacement on the order of 1μm, which according to our results is common in high density beams such as proposed in [1] or [2], is never allowable in VLSI fabrication.

Studying phenomena by computers is very popular and playing an important role in areas of plasma physics etc. Compared with most simulations in such areas, simulations of electron beams are quite easy because the effect of nonlinear terms in the equation of motion is quite small. In this sense, the author would like to emphasize the usefulness of computer study of electron beams.

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