Comparison of Two Computational Methods for High-Velocity Cavitating Flows around Conical Projectiles : OTi-HULL Hydrocode and Two-Phase Flow Method

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Abstract

We present a comparison between two CFD methods for the axisymmetric case: the hydrocode named OTi-HULL and the two-phase flow model presented in Cocchi's thesis. These methods are applied to the calculation of a test case: a biconical body (nose cone with a cylindrical part and rear cone) at a velocity of 3000 m/s in water.

Cocchi's results are comparable to the HULL ones outside the cavity, but inside the cavity the HULL results are wrong in temperature (too high) and in density (too low) compared to Cocchi's.

For the cavity width, the results show an acceptable agreement between the two methods for the cones with half-angles of 45° and 30°. But for the 15° cone there are some discrepancies: the cavity is 10% larger for Cocchi's results in which the phase change of water is taken into account. These results have to be confirmed by experiments.

1. Introduction

ISL has recently begun a computational research activity on underwater ballistics in the transonic domain (velocities from 700 to 1800 m/s). A bibliographical research shows that there are several methods for the computation of cavitating bodies travelling at velocities near the sonic speed in water.

Two methods seemed to be interesting: a two-phase flow model, presented by Cocchi in his thesis (Cocchi (1997); Cocchi et al. (1999)) and a hydrocode named OTi-HULL in use at ISL (Matuska et al. (1991)). These methods use different equations of state and it is interesting to compare the HULL results with Cocchi's method.

The two methods and a comparison of the results obtained for a test case will be briefly presented.

2. The two-phase flow model

This model is based on the temperature equilibrium between the two phases, which means that the vaporization is instantaneous. The transfer terms are unknown, which means that the velocity between the two phases is the same. Cocchi introduces an equivalent fluid with the density given by

$$\rho = \alpha_v \rho_v + (1-\alpha_l)\rho_l \quad (1),$$

with $\alpha_v + \alpha_l = 1$ (2), where α_v and α_l represent the volume fraction of vapor and water, respectively.

The equation of state is written for each phase and for the mixture. For the liquid phase, a modified Tait equation is used with the saturated pressure and density at a given temperature:

$$p = B\left\{\left[\frac{\rho}{\rho_{sat}(T)}\right]^{\Gamma} - 1\right\} + p_{sat}(T) \quad (3),$$

where $B = 3\cdot 10^8$ bar and $\Gamma = 7$.

For the vapor phase, the perfect gas equation is:

$$p = \rho r T \quad (4),$$

with $r = R / M_w$; R is the perfect gas constant and M_w is the water molecular weight.

For the mixture, the pressure and the temperature are the saturated ones obtained by using the equations given in Schmidt (1989), the critical values are T_c = 647.14 K, p_c = 220.64 bar, ρ_c = 332 kg/m^3. An energy equation is also written for each phase.

The EULER equations with the conservative variables ($\rho, \rho u, \rho v, \rho E$) are solved and then the primitive variables ($\alpha_v, \alpha_l, \rho, p, u, v, T$) are obtained with an iterative procedure which determines the temperature for each node and time step, starting from the temperature of the previous time step. More details can be found in the thesis (Cocchi (1997)).

3. The OTi-HULL code

The OTi-HULL code is sold by the Orlando Technology Inc. (Matuska et al. (1991)) and is particularly used in continuum mechanics for impact problems and explosions. We use the Euler module which solves the Euler equations in a rectangular grid without thermal conduction, viscosity and phase change of water.

The equation of state is the Mie-Grüneisen equation given by:

$$p = p_H (1 - 0.5\Gamma\mu) + \Gamma\rho(I - I_0) + p_0 \quad (5),$$

where p_H is the Hugoniot pressure at the density ρ, Γ the Grüneisen coefficient and μ the compression coefficient ($\mu = \rho / \rho_0 - 1$). For water the Hugoniot pressure p_H is given by $p_H = A\mu$, with the constant A given in the material library; Γ is 0.28. I is the internal energy and I_0 is the internal energy for the ambient pressure p_0.

The validity of this equation can be roughly tested in the supersonic domain by comparison with the Tait equation and the agreement is acceptable. In the subsonic domain we can also compare the HULL results with the water properties given in engineering books such as Schmidt (1989) and therefore, we observe an overestimation of the density.

The first tests with the OTi-HULL code applied to supercavitating bodies (Schaffar (1999), (2000), and Schaffar and Pfeifer (2000)) show that this code can be used for the penetration in water: the pressure, density and temperature outside the cavity seem to be correct but the quantities inside the cavity are wrong.

A comparison with the following approximate law giving the velocity decay of the projectile can also be made:

$$V(t) = V_0 / (1 + \alpha V_0 t) \quad (6),$$

with $\alpha = \rho A C_{x0} / 2m$, ρ = water density, A = section of the projectile, C_{x0} = drag coefficient of the cavitator, m = weight of the projectile.

This shows that the HULL code gives values 40% stronger for a cylindrical projectile after the first 200 µs.

In an unpublished work the cavity width was compared with the Levinson (1946) and Serebryakov (1997) (1st order) formulas for a truncated cone projectile at 870 m/s. The HULL code gives a width which is 2 to 2.5 times smaller than the Levinson and Serebryakov formulas. This may not be surprising, because the HULL code is a penetration code and the velocity of 870 m/s is practically the lowest limit of its validity.

4. Computational conditions and tested bodies

The computation is made in an axisymmetrical domain (*x* and *r* are the axial and radial coordinates) with non-reflective conditions at the limits of the domain; in the HULL computation the length of the domain is 100 cm and the radius is 40 cm.

The velocity of the projectiles is 3000 m/s, which means a Mach number of 2 in water. The 3 bodies have the following shape: a nose cone (with half-angles of 45°, 30° and 15°), a cylindrical part and a rear cone (with a half-angle of 45°). The length of the bodies is 3 cm for the 45° angle (mass = 95 g) and 6 cm for the other two (mass of 265 g and 164 g, respectively).

In each case, the projectile is considered to be an « island », which means that it is not distorted during the test. In this case the HULL code also gives the velocity decay and the forces acting on the body. With these values we can compute a drag coefficient which has an unsteady phase and converges very quickly.

In Cocchi's results, a void fraction of 10% is chosen as a cavity limit (density close to 0.9), which can be appreciated more or less precisely in the following figures; further, the cavity half-width is divided by the length of the nose cone. For the two methods the precision of the cavity limits can be estimated at about 10%.

5. Results

Pressure distribution

Figures 1 to 3 show the density distribution at 300 µs for the three projectiles obtained with the HULL code. In these figures the limits of the cavity created by the penetration of the projectile in water are well defined. According to the Mach number of 2, we can also see that the density near the nose of the cone is very high between the bow shock and the projectile. Figures 4 and 5 give the density distribution obtained by the Cocchi method for the 45° and the 15° cones at 175 µs. The comparison of these figures with the HULL results shows that we have the same order of magnitude for the density and the pressure outside the cavity.

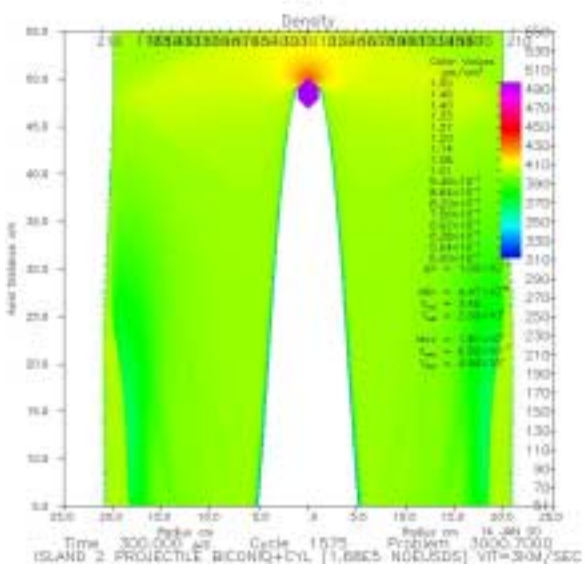

Figure 1. Density at 300 µs for the 45° cone

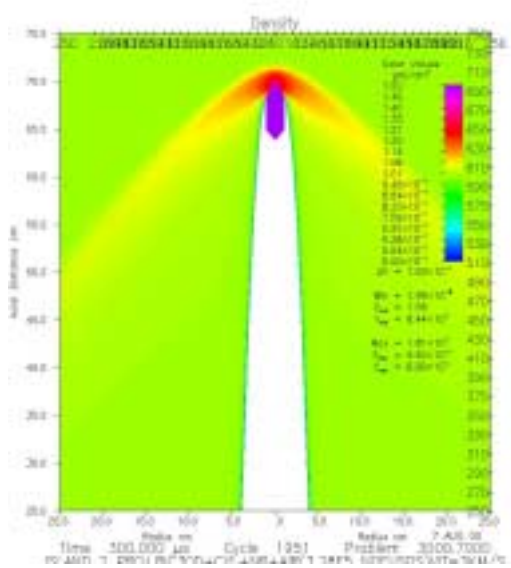

Figure 2. Density at 300 µs for the 30° cone

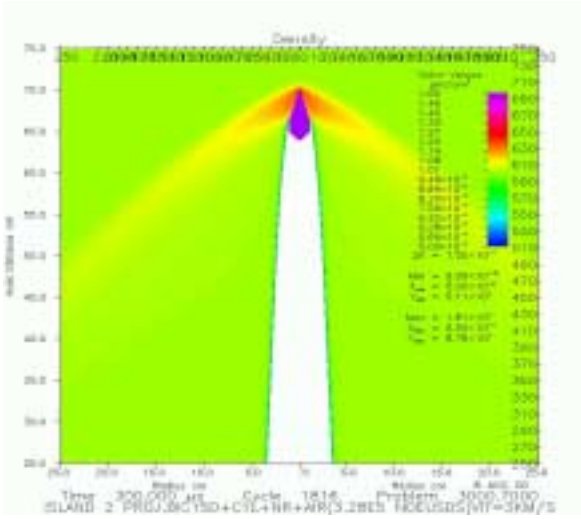

Figure 3. Density at 300 µs for the 15° cone

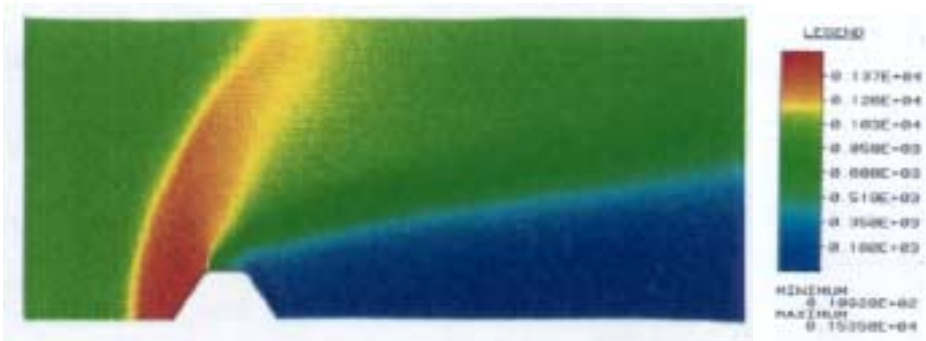

Figure 4. Density at 175 µs for the 45° cone (Cocchi's thesis)

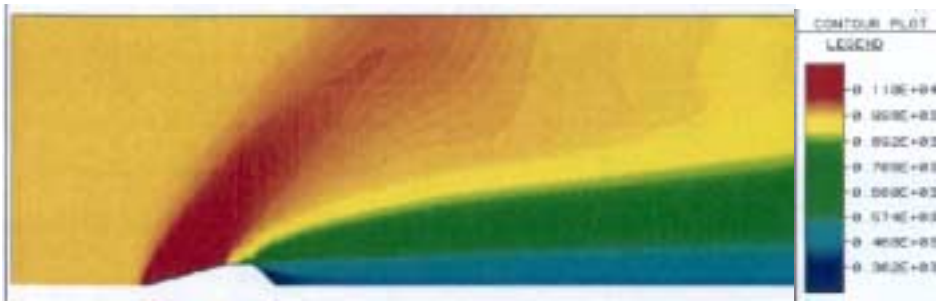

Figure 5. Density at 175 µs for the 15° cone (Cocchi's thesis)

Inside the cavity, it is clear that the HULL results are wrong: the density is too small, the temperature too high and the pressure too low. The reason is obvious: the HULL code does not take into account the phase change of water in the case of too low pressures. On the other hand, Cocchi's results show an increasing water concentration in the cavity near the axis with the diminishing cone angle, which does not seem to be an expected result.

Cavity width

Figures 6 to 8 present a comparison between the HULL results and Cocchi's for the half-width Y of the cavity, normalized by the length L of the nose cone. For the 45° cone, the comparison shows a good agreement between the two methods; for the 30° cone, the results are consistent with one another up to Y/L = 7, and above this value Cocchi's computation provides a larger cavity. For the 15° cone, a discrepancy appears: the Cocchi cavity width is 20 to 25% larger than the HULL cavity, and we cannot explain why.

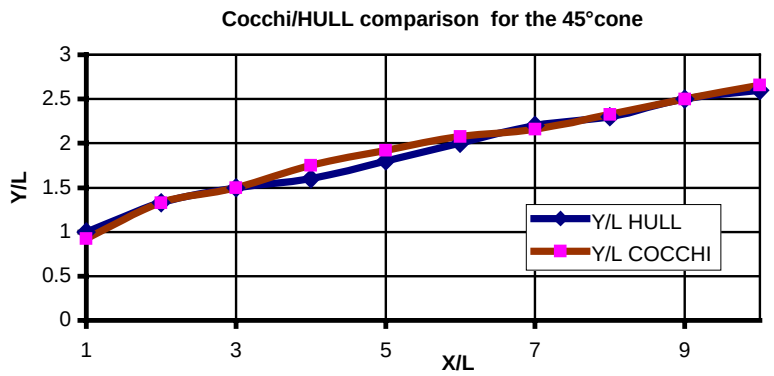

Figure 6. Cocchi/HULL comparison for the 45° cone

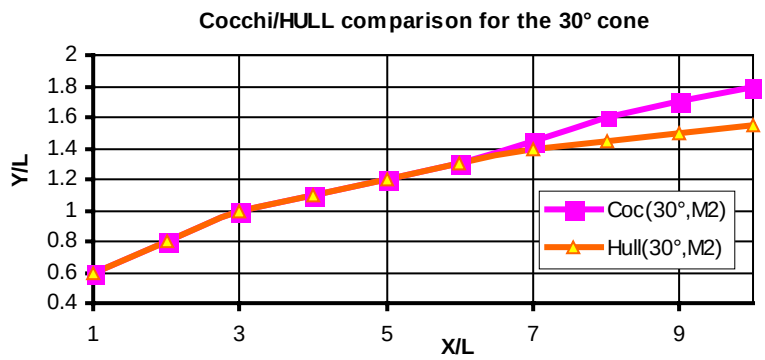

Figure 7. Cocchi/HULL comparison for the 30° cone

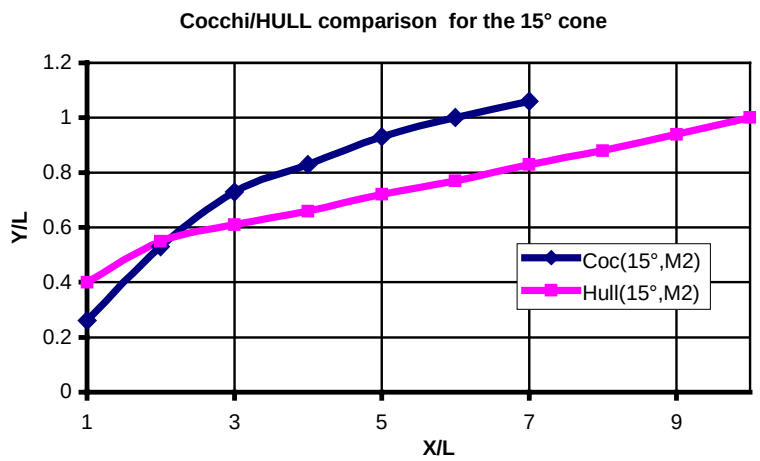

Figure 8. Cocchi/HULL comparison for the 15° cone

Other results

For the 45° cone, the time history of the unsteady drag coefficient is presented in figure 9: after an unsteady beginning, the curve shows a horizontal asymptote near 300 µs with a value of 0.79. Figure 10 provides the time history of the velocity which shows a nearly exponential behavior for the first 100 µs and tends towards an almost linear decay after 200 µs.

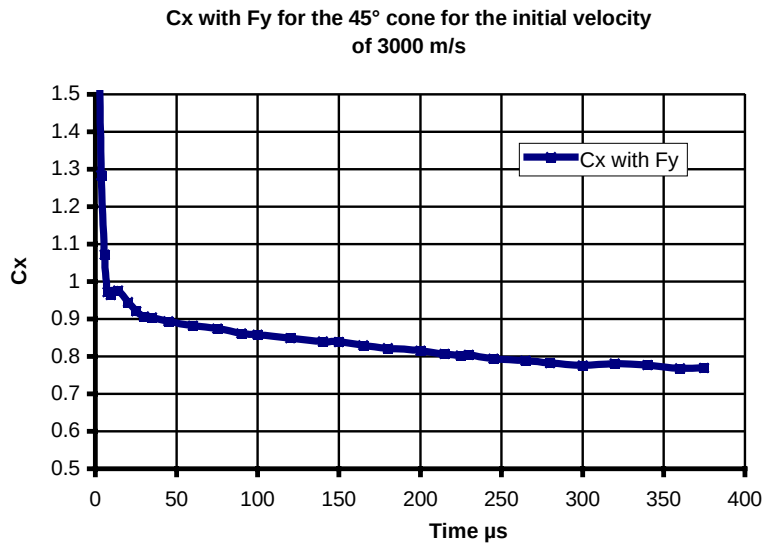

Figure 9. Evolution of the unsteady drag coefficient for the 45° cone

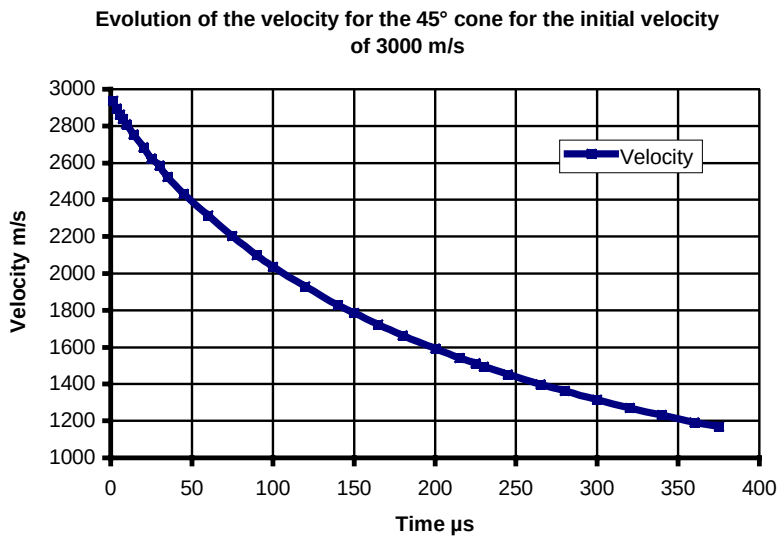

Figure 10. Evolution of the velocity for the 45° cone

For the other two cones, the time histories provide other asymptotic values for the drag coefficient (0.78 for the 30° cone and 0.55 for the 15° cone). These values seem to be larger than those we could extrapolate from the computed values given by Al'ev (1983) for the same Mach number, but for a 26°6 cone (0.55) and for a 19°3 cone (0.35).

In the last two figures some other interesting results obtained with the HULL code are presented. The effect of the velocity is shown in figure 11 in the case of a cylindrical projectile: after an unsteady beginning, all curves show an asymptotic behavior and the value of the drag coefficient increases slowly with the Mach number. The effect of the cavitator diameter is shown in figure 12. For a truncated cone projectile, the diameter of the front disk is varied from 1 cm (diameter of the cylindrical part) to 1 mm (length of the truncated cone: 4 cm for a total length of 10 cm). The drag coefficient decreases very drastically when we normalize with respect to the cylinder diameter; when we normalize with respect to the cavitator diameter, the obtained value is close to 1. The HULL results are also compared with an approximate law giving the drag coefficient as a function of that of a disk: 0.82 multiplied by the ratio $(d_{CAV}/d_{CYL})^2$.

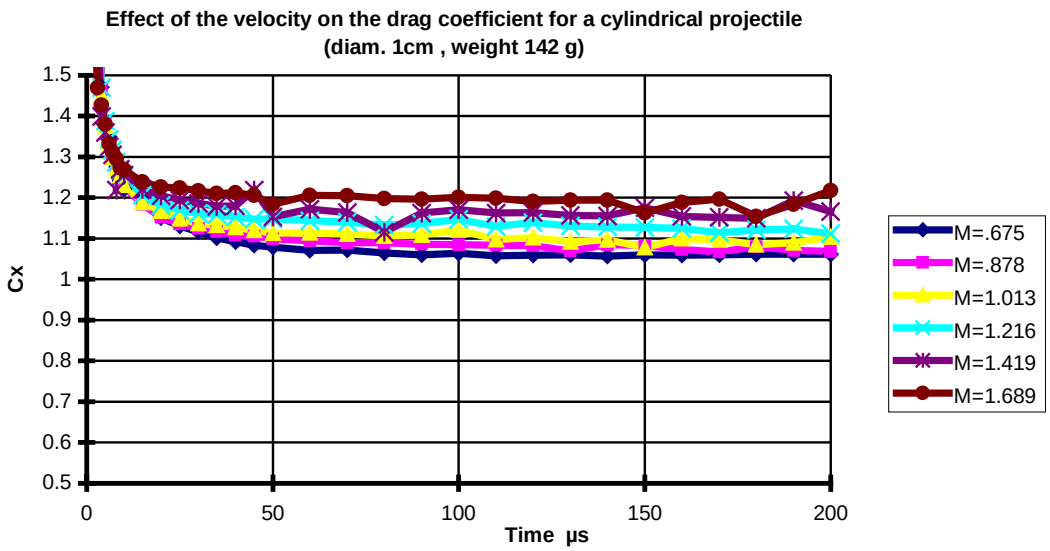

Figure 11. Effect of the velocity on the drag coefficient

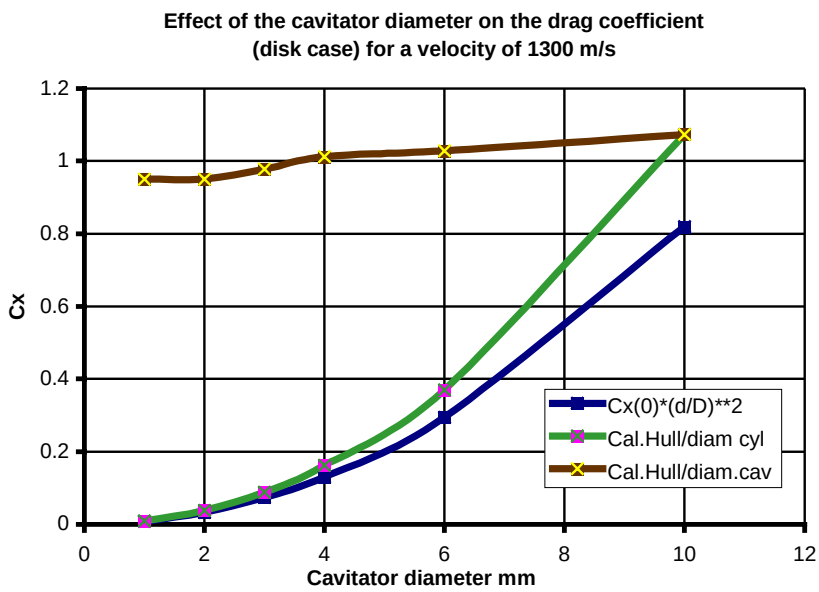

Figure 12. Effect of the cavitator diameter

6. Conclusions

It is found that the two methods give practically the same results outside the cavity, which is not surprising, as the two codes are Euler codes and the equations of state are comparable as demonstrated in Schaffar (1999) for supersonic velocities. Inside the cavity, the HULL code gives wrong results; this is obvious because the phase change of the water is not taken into account. On the other hand, the cavity limits are comparable for two cones (45° and 30°) but very different for the last cone tested (15°), with larger values for the Cocchi method than for the HULL code. Nevertheless, all these results have to be confirmed by experiments in the transonic range for which no results have been published in the literature yet.

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