SOME RESULTS CONCERNED WITH CAVITATION STUDIES IN UNSTEADY HYDRODYNAMICS LABORATORY OF MOSCOW STATE UNIVERSITY

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Abstract

Jet flows in perfect incompressible fluids are analyzed and a number of exact qualitative results for self-similar and limiting flows is given. These general flow properties cannot be, probably, obtained numerically. Only cavitational flows are briefly discussed, more detailed information on the subject can be find in papers Yakimov (1982), (1991), (1992).

1 Introduction

As known, in the general case one can diminish the number of variables in the problem using nondimensional relations between parameters. Besides the flow symmetry, the number of kinematic parameters is also important. Let us recall that in the case of one kinematic parameter composed of governing parameters only three of four independent variables x,y,z,t describe the problem. Such problems are called self-similar Sedov (1959). It is not only the number of variables that is important, but the dependence between their dimensionalities. So the problem of a disk diving (with fluid compressibility taken into account) involves, in addition to the disk velocity V_0, the sound velocity c_0 as well. Their dimensionalities are dependent and the problem is self-similar. As a rule self-similar solutions are asymptotic - for small (or long) time only main parameters remain whose dimensionalities are dependent. In unsteady jet problems including several geometric variables it is convenient to compose dimensionalities of sought-for functions (e.g., a potential or typical velocity) using not only constant parameters, but current time as well.

In what follows self-similar problems for perfect fluids are formulated and experimental modeling the complicated problems is considered.

2 General observations

Consider some general properties of fluid jet flows. On the basis of general ideas on high velocity jet flows in water only the following flow parameters are taken into account: initial density ρ_0, typical pressure on a free surface or far from bodies p_0, pressure inside a cavity p_c, air pressure p_a, kinematic viscosity ν, typical fluid or body velocity V_0, body mass m, typical linear size, (e.g, a cavitator size L or fluid depth H), sound velocity c_0, gravity constant g and density-pressure dependence $\rho = \rho(p)$ (as shown Cole (1950), temperature can be neglected).

Using the listed eleven parameters, one can write (basing on the dimensionality considerations) eight nondimensional parameters

$$Eu = \frac{2p_0}{\rho V_0^2}, \sigma = \frac{2(p_0 - p_c)}{\rho V_0^2}, Re = \frac{V_0 L}{\nu}, Fr = \frac{V_0}{gL}, M = \frac{V_0}{c_0}, \frac{L}{H}, \frac{m}{\rho_0 L^3}, \frac{\rho}{\rho_0} = f\left(\frac{\Delta p}{\rho_0 c_0^2}\right)$$

In what follows the role of the above parameters and their modelling will be discussed for jet flows.

Consider first the important example of high velocity body motion with developed cavitation. Assume that

i - for velocities $V \leq 1000\ m/s$ the liquid density only slightly differs from ρ_0
ii - for $p \approx 1\ atm$ we can write

$$\sigma = \frac{2(p_0 - p_c)}{\rho V_0^2} < Eu = \frac{2p}{\rho V_0^2} \approx 10^{-4}$$

iii - $p \geq 0$ in stagnation regions
iv - outside boundary layers flows are potential, $\Delta\phi = 0$ and the Cauchy-Lagrange integral is valid. The Laplace operator of the integral gives that the pressure is a superharmonic function Birkhoff (1950), Birkhoff and Zarantonello (1957).

$$\Delta p + \frac{1}{2}\rho_0 \Delta(\bigtriangledown\phi) = \Delta p + \rho\Sigma \bigtriangledown u_i \bigtriangledown u_i = 0$$

Thus, as follows from assumption **iv**, in the entire flow region $p > 0$ and the velocity maximum is attained at the cavity boundary.

In real situations $\sigma \neq 0$ and cavities are of high, but finite extension. The analysis of the equation for thin cavities in water (with the influence of cavity edges on their forms taken into account Yakimov (1982), (1983) and Serebryakov (1998)) shows that the form of a cavity fore-part is close to that for asymptotic $\sigma = 0$ and a cavity in the whole is close to an extended ellipsoid. In this case the drag coefficient c_{xm} referred to the midsection of a steady cavity is of the form

$$c_{xm} = c_{xc}\sigma << 1$$

where $c_{xc} \sim 1$ is the cavitator drag coefficient. For example, for $p = 2atm$ (the depth is $10m$) for a body moving at a velocity $1000m/s$ the body total drag in a cavitation flow can be hundred time less than the drag of a body with the same midsection and V_0 in air. The great interest to flows with developed cavitation can be partially explained by the above considerations.

I Let us study a cavity form for $Eu = \sigma = 0$. In this case the body motion is governed by ρ_0, m, L for which only one independent kinematic constant, $[m/\rho_0] = [l]^3$, can be written, therefore the limiting flow is self-similar which means that the cavity form doesn't depend of time Yakimov (1981). For velocity we have

$$(\lambda + m)\frac{dV}{dt} = -\rho_0 c_{xc}\frac{V^2}{2}r_0^2 + T \tag{1}$$

where $L = r_0$ (a cavitator radius), λ is the apparent mass and T is the engine thrust. Integrating expression 1 gives

$$\frac{1}{V} - \frac{1}{V_0} = \frac{p_0 c_{xc}\pi r^2(t - t_0)}{2(\lambda + m)} \tag{2}$$

The same can be written (accurate to a constant) without integrating, but basing on dimensionality considerations, $V = L/t$.

There are many similar flows Figure 1, for example, inertial hydroplaning, body motion in a fluid layer etc. The case $m = \infty$ for finite V corresponds to a stationary flow, $dV/dt = 0$. From expression 2 one can obtain the distance Δs at which the body decelerates from V_0 to V_1

$$\frac{\Delta s}{r_0} = \frac{2(m + \lambda)}{c_{xc}\rho\pi r_0^3}\ln\frac{V_0}{V_1} \tag{3}$$

This formula differs from standard expressions for self-similar flows by the logarithmic function.

As the second example of a jet flow with the same similitude consider fluid expansion due to explosion in a channel of the constant depth H. Then $L = H, m = 0$ and $C_x \rightarrow k$ where k characterizes the momentum removal by a jet Yakimov (1981).

In expression 2 only t_0 (a time moment when the flow can be considered to be formed Figure 2) depends on the explosion power. If $dV/dt \approx g$, then in flows with different explosion powers identical gravity waves arise.

In these examples the solution consists of a sequence of two self-similar solutions and transition to the same groups of unsteady waves. It is of interest to note that the problem on explosion in water was solved earlier than the same problem in gases Sedov (1981).

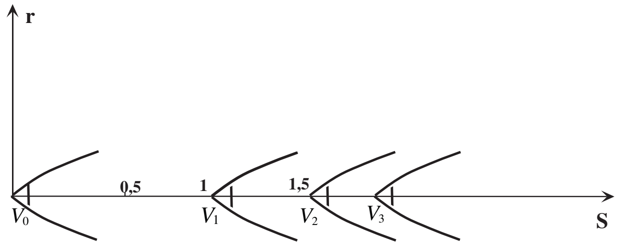

Figure 1

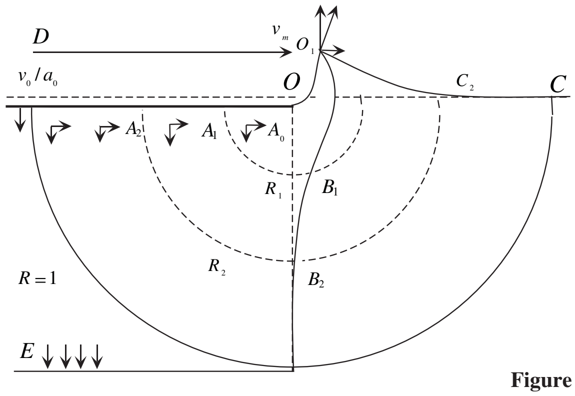

Figure 3

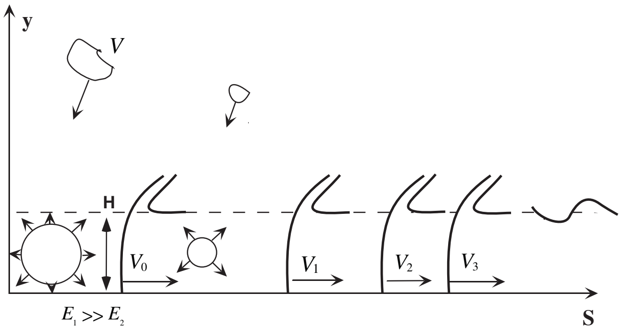

Figure 2

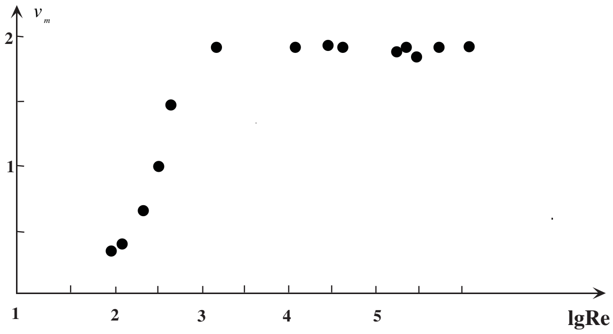

Figure 4

Now, return to the first example see Figure 1. In this example we chose the ideal incompressible fluid model. In practical situations when the velocities are high we must take into account the effect of M on $c_x = c_x(M)$ and on the shape of the cavity. An estimate of the dimensionless coefficient (3) shows that this coefficient can be greater than 10^4 for a body with elongation ≈ 10 and more. For $\sigma = 0$ the asymptotic formula for the radius r of a steady-state cavity in an incompressible has the form Gurevich (1965), Yakimov (1981), (1982).

When $\sigma \ll 1$, $\sigma \neq 0$ a similar shape occurs in the neighborhood of the nose part of the cavity in which the body can be located Yakimov (1983).

$$r = \frac{2xr_0\sqrt{c_x}}{\ln^{\frac{1}{2}} \frac{1}{2}xr_0\sqrt{c_x}}$$

When a body is decelerated in a fluid Figure 1 it is necessary to take into account the inertia forces in the coordinate system moving with the body. In this coordinate system the fluid flows onto the body with a negative acceleration. If for the sake of estimation we assume that each cross-section expands regardless of other cross-sections and far away from the body the pressure is balanced by the same pressure inside the cavity in this cross-section, we obtain that $\sigma = 0$ in each cavity cross-section. Thus, the shape of the cavity coincides approximately with asymptotics for an incompressible fluid [1]

There is no self-similarity for a compressible fluid. When c_x is constant, for $\sigma = 0$ the shape of the steady-state cavity depends on $M < 1$ Yakimov (1981), (1982) and becomes thinner [2]

$$r = \frac{2(1 - M^2)xr_0\sqrt{c_x}}{\ln^{\frac{(1+M^2)}{2}} \frac{1}{2}xr_0\sqrt{c_x}}$$

[1] In Yakimov (1982) there was an error in this asymptotic.

[2] In Gurevich (1965) this dependence, as was shown in Yakimov (1982), was not noted and it was concluded that for $\sigma = 0$ the asymptotics are independent of M.

For example, for $M \leq 2/3$ ($V \approx 1000m/s$) we have $\frac{1}{2}M^2 \leq \frac{2}{9}$ the correction is small within the possible extent of the body. Obviously, at these velocities the dependence of c_x on M is insignificant. Moreover, the corrections to the shape of the steady-state cavity associated with the deceleration and the compressibility have different signs.

II Consider body entry at constant velocity into the perfect incompressible fluid. The first problem of such a type was solved by Wagner (1932) who studied cone or wedge entry into a fluid not necessary normally to its surface and with the body section not necessary circular. In this case $x_i V^{-1} t^{-1}$ are self-similar variables.

Here an impact problem for a disk or a strip of a width $2L_0$ entering the half-space occupied by perfect incompressible fluid is considered. The initial pressure is p_0. In addition to ρ_0, the problem includes two kinematic constants, L, V, therefore in contrast to the Wagner problem formally these flows are not self-similar. However, on edges of such bodies singularities of the type of $V = ia\sqrt{z}$ arise. Here $a = V_0 L^{1/2}\sqrt{2/\pi}$ for a disk and $a = V_0\sqrt{L}$ for a strip. If $V_0 \to 0$ and $L \to \infty$ in such a way that $a \to const$ there are no other singularities in edge vicinities, while initially there are no singularities at all. Hence, for small t flows near edges are described by limiting self-similar solution with variables $x_i a^{-2/3} t^{-2/3}$. For small velocities fluids can be considered to be incompressible and this doesn't violate the similitude. In Figures 2,3 Yakimov (1973), one can see the self-similar nature of the flow in vacuum and jet expanding and closing in the atmosphere due to aerodynamic forces. Birkhoff (1950) pointed out to qualitative influence of small density changes on flow patterns. However, one needs to take into account a jet thickness and its velocity on which both aerodynamic force and inertia of the flexible wing depend.

The more general case occurs when $V_0 \geq c_0$, then the equation of state also involves $c_0, [c_0] = [V_0]$ and there is no linear size and singularity in the flow pattern near the edge Figure 4 Yakimov (1991). Again we have nondimensional parameters of the Wagner type, $x_i c^{-1} t^{-1}$. Figure 4 is taken from the paper by Vasin (1982) where V_m is the jet velocity referred to V_0 and $Re = 2V_0 L/\nu$. In experiments $L = 0.03mm$ and Re is proportional to M. The flow rate through a jet typical base is proportional to V_0 multiplied by an unloading typical length. As follows from the flow pattern, for $3 < \lg Re < 6$ the jet velocity is $V_j = V_0$.

Above two different cases of self-similar solutions for different M were discussed, now consider them in more detail. On impact of any plane body on a surface of compressible (real) fluid the similitude with c_0 does appear. If the pressure-density relation is linear the flow pattern of Figure 3 doesn't change, the ratio V_m/V_0 remains constant, $V_m/V_0 = 2$. However, the relation $\rho(p)$ becomes essential and the above ratio can change as the velocity grows. If $M << 1$ and L is high, the solution with c_0 transits into the self-similar solution with constant a_0 in which jet velocity are high. Evidently, this transition can be seen in Figure 4 Yakimov (1984). Then for $p_0 = p_c$ the inertial deceleration follows. Thus, in this case there is a sequence of three self-similar flows, but not two. For $M < 1$ and small L the transition to incompressible fluid can set in when the body is at a relatively greater depth. There is no solution with the constant a and the transition to self-similar deceleration ensues immediately.

One should note that it appeared to be possible to prepare air bubble-fluid mixture having the same equation of state as water Figure 5 with bubble volume concentration 0.3. The pressure of a few atmospheres in the mixture corresponds to a few hundred atmospheres in water which is of use when modelling body-liquid impact Yakimov *et al.* (1978), Eroshin (1992).

For bubble liquids in a closed vessel of volume Ω the mean pressure can be written as follows

$$\frac{1}{\Omega}\left(p_0 \int_\Omega d\Omega + \frac{dp}{d\rho}\int_\Omega \Delta\rho d\Omega + \frac{d^2p}{d\rho^2}\int_\Omega (\Delta\rho)^2 d\Omega\right) = p_0 + \frac{1}{\Omega}\int_\Omega (\Delta\rho)^2 d\Omega >> p_0 \qquad (4)$$

and on the vessel ends it is much greater due to wave reflection Yakimov (1977)[3].

III. Let us continue to consider developed cavitational flows of **I**, but with gas in a cavity taken into account.

At a cavity end the pulp is formed and washed down by the flow. At a place of pulp formation the equation of state for a medium is sharply changed depending on the gas volume concentration and this results in changing the pressure-density dependence and the Bernoulli integral along streamlines. If bubble slipping is neglected it is obtained under the most general assumptions Yakimov (1974),(1982) that for p_0 the medium velocity in a wake differs from V_0, while the additional force depends only on the flow rate, but not of gas distributions in the wake

[3]This effect is well known to those who open bottles. First they shake wine to prepare mixture and then hit the bottle bottom so that pressure blows cork out

$$F = \frac{p_0 \Omega}{V_0} \ln \frac{p_c}{p_0}$$

where Ω is the volume flow rate referred to p_0. For $p_c > p_0(\sigma < 0)$ we have thrust, while the case $p_c < p_0(\sigma > 0)$ corresponds to drag.

Recently the results on the mechanism of gas washing away were obtained for $\sigma < 0$, this mechanism is clear from Kozlov and Prokof'ev (2001) where one can see the instability of the boundary with small disturbations leading to pulp formation.

Consider gas pressure distributions along cavity boundaries as $V_0 \to \infty$. The analysis shows Yakimov (1982) that gas in the gap between a cavity wall and a body is washed down and in the limit it follows fluid streamlines. Thus, the nonuniform gas injection at different streamlines can lead to different pressures p_c along the cavity boundary.

IV In the Mechanics Institute of MSU a number of installation were developed for experiments concerned with high velocity body motions in water, let us list some of them:

a - channel to study horizontal motions of bodies

b - vacuum channel which makes it possible to model all the above listed nondimensional criteria (except the water compressibility and Re) as well as inclined flows with cavity buoyancy taken into account Sedov (1996) and atmospheres of different gases for different pressures and densities

c - installation to study flow parameters for high velocity body entry into water Yakimov *et al.* (1978), Eroshin (1992).

d - installation to model compressibility for body-water impact Eroshin *et al.* (2001).

e - jet installation to study gas interaction with cavity boundaries etc. Kozlov and Prokof'ev (2001).

3 Conclusions

1. For small cavitation numbers $\sigma \ll 1$ and practically incompressible fluid $\rho = \rho_0$, when the body moves by inertia, there exist a self-similar zone in which the shape of the cavity in the leading part is almost independent of the velocity. Therefore, if the body is inside the cavity, it remains inside the cavity in the process of deceleration. For small depths these conditions correspond to $0.2 \leq M \leq 0.8$.

2. For $M \geq 1$ the fluid compressibility becomes significant and the cavity changes the shape in the process of deceleration. In order for the body to be inscribed into the cavity the velocity should be maintained, for example, by using an engine. Another way to conserve the shape of the cavity in deceleration and for large M is to introduce a controlling shape of the cavitator and its drug.

3. At high velocities the gas ejected at a certain point of the cavity is then carried away along a fluid streamline. Thus, without contact, we can produce transverse forces and moments and a given pressure distribution over the cavity surface.

4. Thin jets are formed in the process of impact of a cavitator on water. The motion of the jets and the entry pattern depend on the atmosphere parameters.

5. When the gas is carried away in the form of a bubbly wake an accelerating or decelerating force (depending on σ) is initiated.

6. There are experimental methods of modelling basic dimensionless parameters.

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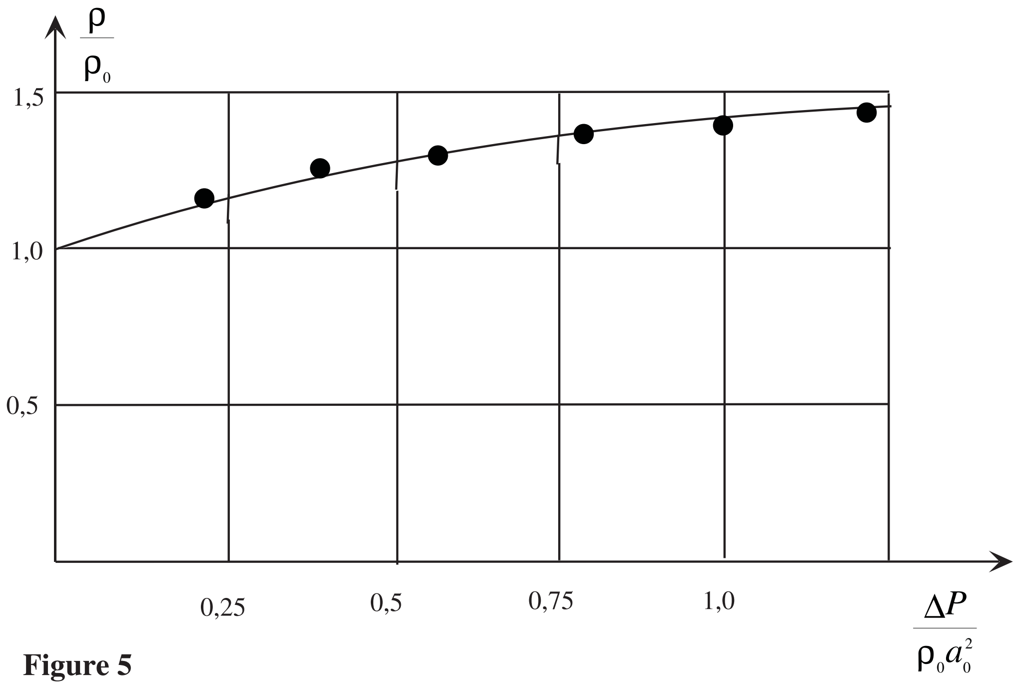

Figure 5