NUMERICAL STUDY OF THE EFFECT OF THE LEADING EDGE SHAPE ON CAVITATION AROUND INDUCER BLADE SECTIONS

O. COUTIER-DELGOSHA, JL. REBOUD (*), R. FORTES-PATELLA

Laboratoire des Ecoulements Géophysiques et Industriels, Grenoble, France (*) LTDS / ENISE, Saint Etienne, France

Abstract

A numerical study of the cavitation behaviour of two-dimensional hydrofoils simulating a section of an inducer blade is presented. Two leading edge shapes were chosen to approach rocket engine inducer designs. They was tested with respect to the development of sheet cavitation.

The numerical model of cavitating flows is based on the 3D code FINE/TURBOTM, developed by NUMECA International. The cavitation process is taken into account by using a single fluid model, which considers the liquid vapour mixture as a homogeneous fluid whose density varies with respect to the static pressure.

Numerical results are compared with experimental ones, obtained in the CREMHyG large cavitation tunnel [Reboud et al. 1996]. Pressure distributions along the foil suction side and the tunnel walls were measured for different cavity lengths. Total pressure measurements along the foil suction side allow characterizing the effects of cavitation on the liquid flow. Influence of the leading edge shape on the cavitation behaviour and comparison between experiments and numerical

Nomenclature

predictions are discussed.

Ср	: pressure coefficient	$= (P - Pref) / (1/2\rho Vref^2)$	(-)
Lcav	: cavity length		(m)
Lref	: reference length	= chord of the foil	(m)
V	: velocity		(m/s)
Vref	: reference velocity	= inlet flow velocity	(m/s)
Р	: static pressure		(Pa)
Pref	: reference pressure	= inlet static pressure	(Pa)
Pv	: vapour pressure		(Pa)
σ	: cavitation number	$= (Pref - Pv) / (1/2\rho Vref^2)$	(-)
ρ	: liquid density		(kg/m^3)

1. Introduction

The prediction of the cavitation behaviour is of first importance for the design of rocket engine turbo-pumps. Indeed, this phenomenon results in strong unsteady forces acting on the pump components, and it may lead to substantial performance losses. The analysis of the vapour occurrence in the pump, as well as the study of the unsteady effects related to cavitation, could directly influence the pump inducer design.

Several studies based on multi-phase flow approaches were developed during the last 10 years [Delannoy and Kueny, 1990, Kubota et al. 1992]. They lead now to the development of three-dimensional Navier-Stokes codes that take into account the cavitation process [Takasugi et al., 1993, Alajbegovic et al., 1999, Kunz et al., 1999, Bunnel and Heister, 2000]. In this context, a numerical model, aiming to take into account the cavitation phenomenon in inducers geometry is developed in collaboration with SNECMA Moteurs and the French space agency CNES. The final objective is to provide assistance to the design and prevision of operating range of rocket engine turbo-pumps, taking into account the steady-state and unsteady effects of cavitation. The model is based on the commercial code FINE/TURBOTM, developed by NUMECA International [Hakimi, 1997]. To simulate the cavitation process, a single-phase flow model is implemented [Coutier-Delgosha et al., 2001a] based on previous studies in 2D [Delannoy and Kueny, 1990, Reboud and Delannoy, 1994, Reboud et al., 1998].

The aim of the present paper is to evaluate the ability of the numerical model to take into account the influence of the leading edge shape on cavitation behaviour in inducer geometries. The study concerns two hydrofoils simulating a section of an inducer blade, held in a cavitation tunnel with two different angles of attack. They are characterised by two different leading edge geometries, chosen to approach rocket engine inducer design [Reboud et al. 1996]. Experimental results are compared with 2D computations in the different cases. A three-dimensional simulation of the complete hydrofoil geometry is also performed in one case, to illustrate the 3D effects on the cavitation sheet.

2. Experiment

The experimental study of the cavitation behaviour of two-dimensional hydrofoils simulating a mid-span section of an inducer blade has been performed in the large cavitation tunnel of the CREMHyG laboratory (Figure 1) [Reboud et al. 1996].

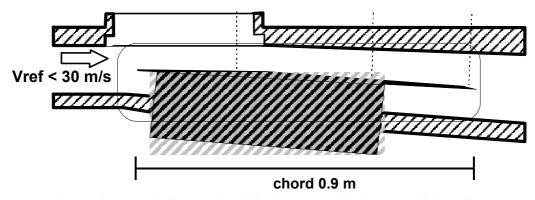


Figure 1: large cavitation tunnel (width = 0.12 m) and inducer type blade section.

The upper and lower walls of the cavitation tunnel have been designed to generate along the suction side of the foil a pressure distribution as close as possible to the one existing along the inducer blade in non-cavitating condition. The hydrofoil chords are 0.9 meter long and their relative thickness is smaller than 1.5%. A 5 mm thin streamlined leg, through which pressure taps have been drilled, holds the foils. The reference flow velocity Vref and static pressure Pref are measured in the upstream rectangular section Sref ($137x120 \text{ mm}^2$), located 200 mm upstream the foil leading edge. Those reference values are used to define the cavitation number σ . The maximum inlet flow velocity Vref may be larger than 30 m/s. No marked effect of the amount of air dissolved in water has been observed on developed cavitation. During the present experiments, the air content of the water has been kept near saturation.

The behaviours of two different leading edges have been compared with respect to the development of sheet cavitation. Their shapes are reported on Figure 2. The first leading edge presents a parabolic shape, about 3 mm long, creating with the suction side a sharp ridge. It simulates a blade whose upstream end has been bevelled. The second one is sharper and its rounded upstream end, where the cavity detachment occurs, presents a small curvature radius close to 0.2 mm. The two profiles will respectively be named hereafter "bevelled leading edge" (BLE) foil and "sharp leading edge" (SLE) foil. The second hydrofoil is 3 mm shorter than the first one and its relative thickness is a little smaller, but their two suction sides are identical.

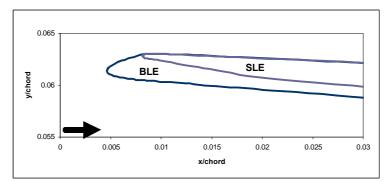


Figure 2. BLE and SLE leading edge shapes.

Two different angles of attack have been tested: 4.1 and 4.5 degrees. They have been chosen to provide flow conditions close to those predicted in a blade-to-blade channel of the inducer at its nominal operating point. Because of the small ratio between the tunnel height and the chord length, the 0.4 degrees difference has a great influence on the cavitation behaviour.

Pressure measurements have been performed along the foils suction side (10 taps) and the tunnel walls (3 taps in the reference section, 7 along the lateral and lower walls and 4 along the upper wall). Pressure taps are connected with 1 mm diameter Rilsan pipes to piezo-resistive transducers with accuracy better than +/- 0.1% of the 5 bar range.

The flow rate is measured by an electromagnetic flow meter, with a relative accuracy of 0.2 %. The combination of inaccuracies on pressure and velocity measurements ensures on the pressure coefficient Cp and on the cavitation number σ a global uncertainty of +/- 0.02 at Vref = 10 m/s.

Total pressure measurements have been performed using a 4 holes probe along 3 axes crossing the upper channel at 37%, 70% and 95% of the chord from the leading edge (Figure 1). Velocities have been deduced from the total and static pressure measurements. Comparisons with LDA data acquired along the same axes were used to validate the method under non-cavitating and cavitating conditions.

3. Physical model

Two-phase flow models are based on the assumption that the fluid is present in the computational domain both as liquid and vapour. The vapour is characterized by the density ρ_v and the liquid by the density ρ_l . On each cell of the mesh, the unknowns are calculated for each phase, by averaging them on the volume occupied respectively by liquid and gas [for example: Ishii, 1975].

In the present work, we apply a single fluid model based on previous numerical and physical work developed in LEGI [Delannoy and Kueny, 1990, Reboud et al. 1998]. It considers only one fluid, characterized by a density ρ that varies in the computational domain according to a state law. When the density in a cell equals the liquid one ρ_l , the whole cell is occupied by liquid, and if it equals the vapour one ρ_v , the cell is full of vapour. Between these two extreme values, the cell is occupied by a liquid/vapour mixture that we still consider as one homogeneous single fluid. The void fraction $\alpha = (\rho - \rho_l) / (\rho_v - \rho_l)$ corresponds to the local ratio of vapour contained in the mixture. In this simple model, the void ratio α is related to the state law, the fluxes between the phases are treated implicitly, and no supplementary assumptions are required.

Moreover, concerning the momentum fluxes, our model assumes that locally (in each cell), velocities are the same for liquid and for vapour: in the mixture regions gas structures are supposed to be perfectly carried along by the main flow. This hypothesis is often assessed for this problem of sheet-cavity flows, in which the interface is considered to be in dynamic equilibrium [Merkle et al. 1998, Song and He, 1998, Kunz et al. 1999]. The momentum transfer between the phases is thus strongly linked to the mass transfer.

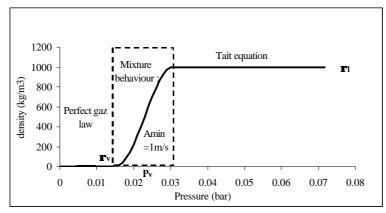


Figure 3: The barotropic state law $\rho(P)$

In the present work the empirical barotropic law $\rho(P)$ used is presented in Figure 3. When the pressure is respectively higher or lower than the vapour pressure, the fluid is supposed to be purely liquid or purely vapour, according respectively to the Tait equation and to the perfect gas law. The two fluid states are joined smoothly in the vapour-pressure neighbourhood. It results in the evolution law characterized mainly by its maximum slope $1/A_{min}^2$, where $A_{min}^2 = \partial P/\partial \rho$. A_{min} can thus be interpreted as the minimum speed of sound in the mixture.

4. Numerical model

To calculate cavitating flows we apply the commercial code FINE/TURBOTM developed by NUMECA International. It is a three-dimensional structured mesh code that solves the time dependant Reynolds-averaged Navier-Stokes equations. A detailed description of the code is given in [Hakimi, 1997].

Unsteady Calculations

Time accurate resolutions use the dual time stepping approach (see [Jameson, 1991]). Pseudo-time derivative terms are added to the equations. They march the solution towards convergence at each physical time step. The code resorts to a multigrid strategy to accelerate the convergence, associated with a local time stepping and an implicit residual smoothing. This kind of resolution is devoted to highly compressible flows. In the case of low-compressible or incompressible flows, its efficiency decreases dramatically. This well-known problem has been addressed by many authors and solved by introducing a preconditioner [Hakimi, 1997]. This one is based on the studies of [Turkel, 1987, Choi and Merkle, 1993]. It consists in multiplying the pseudo-time derivatives by a preconditioning matrix Γ^{-1} . Such modifications have no influence on the converged result, since these terms are of no physical meaning, and converge to zero.

The discretization is based on a finite volume approach. We use a second order central scheme, which must be associated with two artificial dissipation terms, respectively of second and fourth order. The first one is activated in the strong pressure gradient areas, such as shocks. The other one is used in the whole domain, and it results in a second order space accuracy. The pseudo-time integration is made by a four-step Runge-Kutta procedure.

The physical time-derivative terms are discretized with a second order backward difference scheme that ensures a second order accuracy in time.

This numerical model was adapted to treat the cavitation process. The key point of this adaptation is the modification of the state law of the fluid. Our barotropic law implies the simultaneous treatment of two different cases: the fluid is highly compressible in the two-phase flow areas (the Mach number can be as high as 4 or 5) and it is almost incompressible in the pure vapour or pure liquid areas. So the main difficulty consisted in managing these two different states of the fluid, without creating any spurious discontinuity in the flow field. Besides, the cavitation consists in a very sharp and very rapid process. It means that the density time fluctuations and the density repartition must be smoothed, to avoid numerical instabilities. Thus, we use an under-relaxation of the variations of the density to prevent too fast changing during one pseudo-time step. We also modified the switch between the two types of artificial dissipations. The second difference term is now also activated in the strong density gradient areas, such as cavitation sheet interface. Turbulence Model

We use for the calculations presented in this paper a classical Baldwin-Lomax turbulence model. Other calculations considering k- ε turbulence models are in progress to improve the physical analyses. A more detailed study of the influence of the turbulence model on unsteady cavitation simulation is proposed in [Coutier et al., 2001b]. It shows a major effect of the compressibility effect modelling on the unsteady behaviour of cloud cavitation.

Boundary Conditions

The boundary condition setting is based on a system of dummy cells. Classical incompressible types of boundary conditions are applied: imposed velocities at the inlet, and an imposed static pressure at the outlet. Numerical studies have been started in LEGI to improve these conditions, mainly by taking into account the test rig influence [Longatte, 1998], but have not been applied in the cavitating case yet. Laws of the wall are applied along solid boundaries. Initial Transient Treatment

First of all, a steady step is carried out, with a pseudo vapour pressure low enough to ensure no vapour presence in the whole computational domain. Then, this vapour pressure is increased progressively during the early time steps, until the required cavitation number σ is reached. Vapour appears progressively in the low static pressure regions during this transient. The cavitation number is then kept constant throughout the computation.

5. Two-dimensional results

2D computations were performed on both geometries, for the two experimental angles of attack of 4.1 and 4.5 degrees. A 10.000 cells mesh was applied in the four cases (Figure 4), and a large range of cavitation numbers was considered, according to the experimental results. In the case of the SLE foil at 4.5 degrees incidence, a second mesh has been tested, more refined near the leading edge.

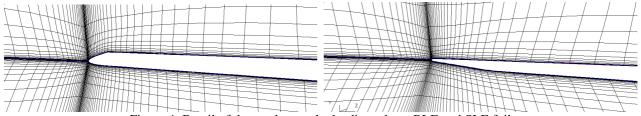


Figure 4: Detail of the mesh near the leading edges: BLE and SLE foil

In all computations a cavitation sheet appears on the suction side of the foil and completely stabilises as soon as the operating cavitation number is kept constant. Then, decreasing the cavitation number σ in successive steps allows drawing the law: cavitation number σ versus cavity length. Figures 5a and b present the results for the two angles of attack, respectively in the case of the BLE and SLE foils. Experimental similar results are also provided on these figures, to enhance comparison. We obtain a reliable agreement between numerical and experimental results, the case of the SLE foil at 4.5 degrees angle of attack needing nevertheless at particular treatment, as discussed hereafter.

4.1 degree angle of attack: experiments and computations confirm that the leading edge shape has almost no influence on the cavities extend at low incidence. Visualisations under stroboscopic light show smooth and quasi-steady cavitation sheets at all the cavitation numbers tested. Only a short fluctuating wake downstream of the cavity is composed of small travelling vapour structures.

4.5 degree angle of attack: experimental observations report the unsteady behaviour of the cavities on the two foils, and a strong influence of the leading edge shape. At given cavitation number σ , the mean cavity length is about two times larger on the SLE foil than on the BLE foil, and correspondingly for the same cavity length the cavitation number is about

25% higher. This effect was first not obtained in the numerical simulations while making the computation by decreasing progressively the cavitation number: contrarily, it can be observed on Figure 5b that very short cavities are then predicted in the case of the SLE foil. That is why the computation was continued, re-increasing the cavitation number from the lowest value. A strong hysteresis phenomenon is then observed, the cavity development remaining much larger during that second phase.

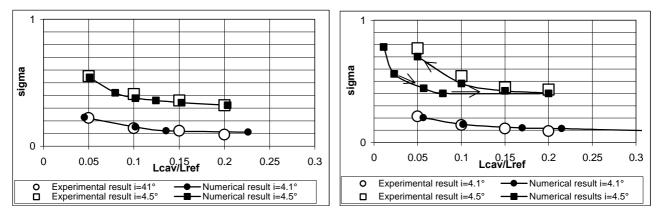


Figure 5. Cavitation number σ vs. (Lcav/Lref) laws: a) BLE foil; b) SLE foil* (*at 4.5 degree, the arrow shows the succession of calculations)

It can be seen on Figure 6a that the pressure distribution obtained by the calculation along the suction side is the same on BLE and SLE foils, after the first 2% of the chord. The minimum of pressure, very close to the leading edge, is a little lower on the SLE foil. The velocity fields calculated in non-cavitating condition are illustrated by Figure 6b. An area of separated flow can be seen in the case of the SLE foil. According to numerical simulations, the separated flow is progressively filled with vapour when the cavitation number is decreased. For low value of σ , the cavity length becomes much larger than the initial separated flow structure and breaks it. While increasing again the cavitation number the cavity length remains larger than in the previous phase, and the agreement with experiment becomes much better.

In fact, the hysteresis phenomenon is not observed experimentally. The hysteresis predicted by the numerical calculations can be related to the physical and numerical models that do not well describe the shear effects of separated flows and do not capture the unsteadiness of the cavitation region. Indeed, from the experimental point of view the cavities adopt on the two foils at 4.5 degree angle of attack an unsteady behaviour associated with large vapour cloud shedding. The numerical model fails to simulate this behaviour, and on the contrary completely stabilises the cavity. This current limitation of the model could be linked to the use of standard turbulence model [Coutier-Delgosha et al. 2001b]. Work is in progress to qualify the different effects of the model used and of the numerical scheme, to improve the simulations of unsteady cloud cavitation behaviours.

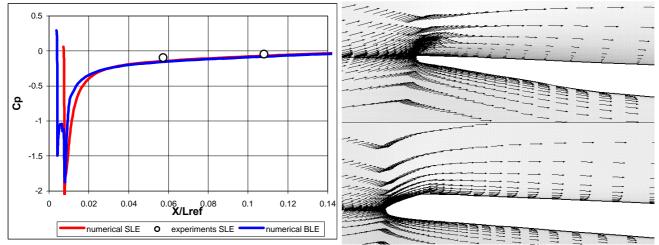


Figure 6. Comparison of numerical pressure distributions (a) and leading edge flow fields (b) for the two hydrofoils in non-cavitating conditions.

The detachment of the cavitation sheets obtained in the four cases is shown on Figure 7, for the cavitation number corresponding to a cavity length close to 15% of the chord. It can be noticed that the high angle of attack leads to a faster growing of the cavity thickness downstream of the detachment point, and to a larger void ratio along the foil surface.

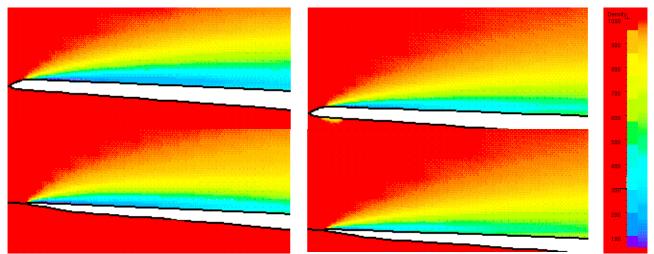


Figure 7: shape of the cavities with Lcav = 15% Lref a) Bevelled leading edge $-i=4.5^{\circ}$ b) Bevelled leading edge $-i=4.1^{\circ}$ c) Sharp leading edge $-i=4.5^{\circ}$ d) Sharp leading edge $-i=4.1^{\circ}$

Quantitative comparisons are investigated between experiments and the numerical results in the case of the bevelled leading edge at high angle of attack. The static pressure evolution along the profile is represented in Figure 8 for a cavity length increasing from 5% of the chord to 20% of the chord.

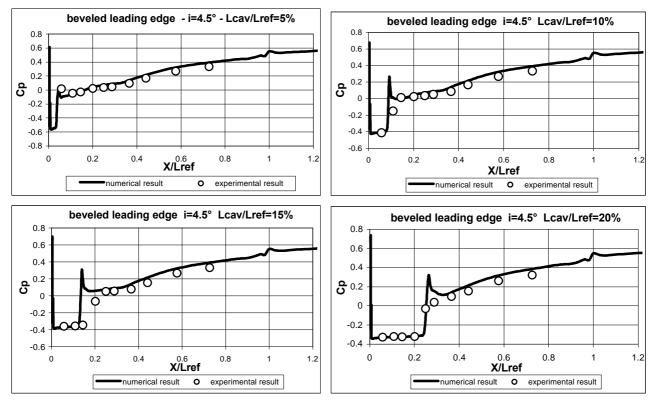


Figure 8: Static pressure distribution on the profile: numerical and experimental results (BLE foil, Vref = 16m/s).

We observe a general good agreement between the numerical results and the experimental data: the charts confirm that the cavity length is correctly predicted. The pressure distribution downstream from the closure area is a little over estimated by the calculation, which is mainly an effect of the 2D simulation that do not take into account the obstruction effects linked to the boundary layers developing along the sides walls of the tunnel. Nevertheless, the most important discrepancy concerns the pressure increase in the cavity closure area, which appears significantly more pronounced in the calculations than in the experiments. This sharp elevation predicted by the model in the cavity wake can be related to the complete stabilization of the cavitation sheet. The real behaviour involves self oscillations of the cavitation sheet and the measured time-averaged pressure distributions downstream to the cavities appear smoothed.

Velocity profiles across the upper channel are more closely studied. Three axes almost perpendicular to the foil suction side, respectively at 37%, 70%, and 95% of the chord length were investigated by means of a 4 holes probe. The component of the velocity vectors normal to these sections is represented in Figure 9 in non cavitating and cavitating conditions. The numerical results and the experimental data are compared. Because the obstruction effect due to the boundary layers along the side walls and along the leg holding the foil is not taken into account in the 2D simulation, the experimental mean velocity in the 3 sections are about 10% higher than the numerical ones. That is why to compare the profiles, all values have been divided by the mean velocity in non cavitating condition.

In non cavitating conditions, the boundary layer appears almost symmetric on both sides of the channel. In cavitating condition, the important increase of the displacement thickness due to the presence of the cavity wake is clearly visible all along the foil suction side. The interaction between that wake and the flow at the foil trailing edge leads to the slight decrease of the flow rate passing in the upper channel in cavitating conditions, what modifies a little the flow circulation around the foil and the resulting lift [Reboud et al. 1996]. The agreement between measurement and simulation is satisfactory, in non cavitating and cavitating conditions. The quantitative influence of the cavitation sheet on the suction side boundary layer is notably well predicted.

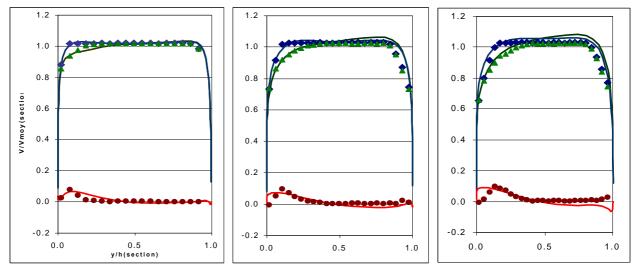


Figure 9. Velocity profiles in three sections located respectively at 37%, 70%, and 95% of the chord. All values are divided by the mean velocity in the corresponding section in non cavitating condition. (BLE foil at 4.5 degree, Vref = 16m/s). Non cavitating condition experimental points \diamond ; numerical result (full line); Cavitating condition (Lcav/Lref = 15%) experimental points ; numerical result (full line) ; Difference between non-cavitating and cavitating conditions experimental \diamond ; numerical result (full line).

6. Three-dimensional computation

A three dimensional calculation taking into account the complete experimental geometry was performed, to illustrate the 3D ability of the numerical code and to estimate the effect of 2D simplifications applied in the previous cases. A 1000000 cells mesh of the SLE foil at 4.1 degrees angle of attack is used. The cavitation number is fixed equal to 0.2, which leads to a cavity length close to 5% of the chord. The cavity shape, shown in Figure 10, appears clearly 3D, with an important influence of the side walls boundary layers, also visible on the streamlines drawn from the foil leading edge.

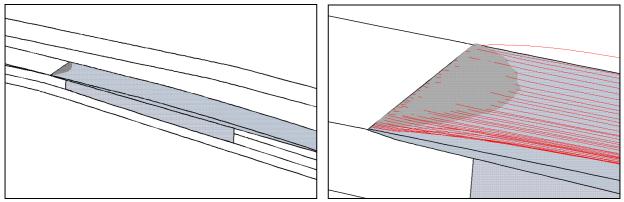


Figure 10. Cavity shape: SLE foil, $\sigma = 0.02$. Grey surface corresponds to a void ratio of 5%. Streamlines of the flow in the vicinity of the cavity surface are drawn in red.

7. Conclusion

A numerical study of the cavitation behaviour of two-dimensional hydrofoils simulating a section of an inducer blade was presented. Two dimensional computations were performed in different cases: two different foil leading edge shapes, two different angle of attack. A general good agreement with experiments is obtained for the cavity length at different cavitation numbers. In one case, namely the sharp leading edge foil at the higher angle of attack, the presence in non cavitation conditions of a separated flow near the sharp leading edge creates from the numerical point of view the condition of a strong hysteresis phenomenon. Only the simulation performed by increasing the cavitation number from its minimum value provides a behaviour that conforms to experimental observations. With this angle of attack, the cavity extend is larger than with the bevelled leading edge foil, in both experiments and simulations. The self oscillating behaviour of cavitation is not simulated at the present stage of the model. There is no effect of the leading edge shape at the lower angle of attack. Static pressure distributions along the foil suction side and velocity profiles in the cross sections of the foil suction side channel agree well with measurements. The presence of the cavitation sheet leads to the modification of the boundary layer all along the foil, with a probable effect on the trailing edge flow. A first 3D computation shows the influence of side wall boundary layers on the aspect of the attached cavity. Another application of the model, to 3D cavitating flows in a centrifugal pump, is presented in [Hofmann et al., 2001].

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