SOME MODELS OF PREDICTION OF SUPERCAVITATION FLOWS BASED ON SLENDER BODY APPROXIMATION

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Abstract

Paper contain results o f development of approximate methods and analysis of possibilities of flows and motion predictions for high speed movement prolate mainly axisymmetric bodies in water with supercavitation. Investigations are based on known approximation of the Slender Body Theory (SBT) and application of Matched Asymptotic Expansions Method (MAEM), integral conservation laws , heuristic models and another approaches. Alike consideration give the possibility to analyze the number of key problems as whole from united point of view. With account of two the most important ranges of speeds: moderate high speeds and very high speeds in the range of Mach Number M ~0.5-2 and over it is applied accordingly two base models: of ideal incompressible fluid and izentropically compressible fluids. Every of this characteristic cases correspond different cases of applications and accordingly developed parts of the theory. The problems of development of the methods for prediction of steady and unsteady prolate supercavities for given pressure and cavities with gas injection are analyzed with account of base perturbations factors. Possibilities and state of problem of prediction supercavitation for very high speeds with account of compressibility effects are considered. Peculiarities and possibilities of predictions movement trajectory of supercavitating bodies are analyzed and influence of hydro elastic effects under motion. are investigated

1. Introduction

Applying of the supercavitation give the possibility isolated moving body of water to avoid considerable viscous forces. In doing so the least cavitational drag C_D for cavity middle section (body compactly enough inserted in the cavity) are reached namely for the slender cavities. Possibilities of decreasing of C_D are limited by maximal possible bodies aspect ratios which can provide it's strength under motion. Really for high speed movement in water values of $C_D \approx 0.05 \div 0.001$ are possible and for some cases it is possible to reach the values comparable as in the air. For very high speeds in water what are realized with help launching of not large bodies by mass 0.1-0.5кг it is possible to reach in water considerable ranges comparable as in the air. Process of high speed motion of prolate supercavitating body can be illustrated by simple model. In case of slender cavity cavitator is small enough and it's drag practically is not depending on cavity form and cavity form is not depending on cavitator form and is defined by it's drag only. Moving cavitator push motionless fluid aside and its drag is transformed to the kinetic energy of forming near cavity practically radial flow in the every motionless section of fluid passed through by cavitator. Further we have practically independent expansion of the cavity section in the motionless fluid by inertia under action pressure difference in the flow and cavity. Body movement can take place in the finite cavity or in the forward part of very large cavity with small gap between cavity and body surfaces practically without contact body and fluids but for hydrodynamic interaction not large wetted back part of body for providing of movement stability. Realization of supercavition motion is connected with several essential points. For moderate speeds this is gravity influence what can considerably deform cavity and this fact can be important from point of view of flow in the back part of cavity defining gas losses processes. Possibility of considerable cavity deformation under gravity limit supercavitation application for moderate speeds till 100-200m/s for not large vehicles only. Another essential point here is necessity of applying of gas injection in order to have prolate enough cavity for really essential pressure which could provide small enough C_D. For very high speeds problems of water compressibility effects and problems connected with possible appearance of extremely high hydrodynamic strengths and necessity to take into account stresses deformation body state become especially essential. Purpose of the consideration is attempt to have analyzed possibilities of the theory as whole apply to the most part of the basic problems in the most interesting range of Mach Number moderate and very high speed from definite united point of view.

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2. Base equations

The most characteristic is statement of the problem for the steady potential flow with the free boundaries in the unlimited volume of ideal incompressible fluid with Riabyshinsky scheme of closure. Cylindrical coordinate system r, x is used. $r = r_1(x)$ given cavitator form, ΔP given pressure difference in the flow and cavity. Cavitator and cavity are considered as whole as some slender body with small slenderness parameters. $\delta \sim 1/\lambda$ (λ-aspect ratio). For $\delta \to 0$ with accuracy of $\delta^2 \ln 1/\delta^2$ on base known (SBT) expansion by M. Adams and W. Sears, F. Frankl - E. Karpovich integer- differential equation (IDE) for slender cavity behind slender cavitator is (1973):

$$\underset{(\ln 1/\delta^2)^{-1}}{\frac{1}{2R^2}\left(\frac{dR^2}{dx}\right)^2} + \underset{(1)}{\frac{d^2R^2}{dx^2}\ln\frac{\beta^2 R^2}{4x(L-x)}} - \underset{(\ln 1/\delta^2)^{-1}}{\int_0^{x_o}\frac{\frac{d^2r_1^2}{dx^2}\Big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1 - x|}dx_1} - \underset{(\ln 1/\delta^2)^{-1}}{\int_{x_o}^{L}\frac{\frac{d^2R^2}{dx^2}\Big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1 - x|}dx_1} - \underset{(\ln 1/\delta^2)^{-1}}{\frac{\frac{dr_1^2}{dx}\Big|_{x=0}}{x}} + \underset{(\ln 1/\delta^2)^{-1}}{\frac{\frac{dR^2}{dx}\Big|_{x=L}}{L-x}} = \underset{(1)}{2\sigma},$$

$$[R = r_1(x)]_{x=x_o} \qquad \left[\frac{dR^2}{dx} = \frac{dr_1^2}{dx}\right]_{x=x_o} \quad (2.1a) \qquad \left[R^2 = 0\right]_{x=L} \quad (2.1b) \qquad (2.1)$$

Here $x = 0$ is cavitator nose, $x = x_0$ fixed separation section , $\sigma = \dfrac{\Delta P}{\rho U_\infty^2}$, U_∞ flow speed., closure is supposed as point $x = L$ (automatic closure on some small body). For incompressible $\beta = 1$, $\beta^2 = |1 - M^2|$. Orders of the terms are indicated below. Not stationary variant of IDE (1978) in the coordinate system connected with motionless fluid is:

$$\frac{1}{2R^2}\left(\frac{\partial R^2}{\partial t}\right)^2 + \frac{\partial^2 R^2}{\partial t^2}\mathrm{Ln}\frac{R^2}{4[x_n(t)-x][x_c(t)-x]} - \int_{x_n(t)}^{x_s(t)}\frac{\frac{\partial^2 r_1^2}{\partial t^2}\Big|_{x=x_1} - \frac{\partial^2 R^2}{\partial t^2}}{|x_1 - x|}dx_1 - \int_{x_s(t)}^{x_c(t)}\frac{\frac{\partial^2 R^2}{\partial t^2}\Big|_{x=x_1} - \frac{\partial^2 R^2}{\partial t^2}}{|x_1 - x|}dx_1 - \frac{dx_c}{dt}\frac{\frac{\partial R^2}{\partial t}\Big|_{x=x_c(t)}}{|x_c(t)-x|} + \frac{dx_n(t)}{dt}\frac{\frac{\partial r_1^2}{\partial t}\Big|_{x=x_n(t)}}{|x_c(t)-x|} = \frac{4\Delta P}{\rho} \qquad (2.2)$$

t - time, $x_n(t), x_s(t), x_c(t)$ - motion laws of nose, separation section, cavity end.

3. Short review of some results

For the first steps 40-50years the most effective in the supercavitation field was applying of namely simple heuristic models, integral conservation laws, perturbations methods. Starting from known investigations by H. Reichardt, G. Birkhoff the ellipsoid cavity form and known dependence for the maximal cavity radius R_k (3.1) were defined (R_n - cavitator radius, c_d - cavitator drag coefficient, $k \sim 0{,}94 - 1$) and was understood practical independent of prolate cavity sections expansion what than was expressed by G. Logvinovich with help the principle of cavity independent expansion. Note here known investigations by A. Armstrong, M. Plesset, L. Woods, J.-M Michel, A. May …Very effective was applying of flat solutions for describing of axisymmetric flows, asymptotic approach here was developed by P. Garabedian (1956) obtained known dependence for cavity aspect ratio λ (3.22). Note known 2-term asymptotic of streamline expansion at infinity $x \to \infty$ N. Levinson (1947), M. Gurevich (1947 (3.3)).

$$R_k = R_n\sqrt{\frac{c_d}{k\sigma}} \quad (3.1) \qquad \lambda^2 = \frac{\ln 1/\sigma}{\sigma} \quad (3.2) \qquad R^2 = 2\sqrt{c_{do}}\frac{x}{(\ln x)^{0,5}}\left[1 - \frac{1}{4}\frac{\ln\ln x}{\ln x} + \cdots\right] \sim \frac{x}{\ln x^{0.5}} \quad (3.3)$$

Creating of known linear flat supercavitation theory by M. Tulin (1964) considerably stimulated creating of alike linearized axisymmetric theory. Here numerical- analitical methods of prediction on base (IDE) alike as (2.1) were developed by T. Nishiyama and H. Kobayshi (1969), C. Chou (1974), W. Vorus (1986).... Very effective here is asymptotic approach on base (SBT) S. Grigorian (1959), Yu. Yakimov(1969-83), V. Serebryakov (1972-91), A. Petrov (1986). Thank to very complicated structure of solutions considerable difficulties were for developing of nonlinear numerical calculation theory. At present there are here good enough developed theory mainly for steady flows, note here known investigations by C. Brennen (1969), (Л. Гузевский (1975-79). Ideal incompressible fluid is one of the basic supercavitation model, but nevertheless this model is not enough for many of key effects connected with problems of initial cavitation, appearing and developing of two phase zones in the flows, phases changes in the flows, viscosity influence… which at present are one of the main directions of modern investigations: R. Arndt, A. Arakery M. Billet, C. Brennen , S Ceccio, H. Kato, J. Levkovsky. Thank to restricted enough field the review is very not complete, more detailed there are in known reviews by R. Arndt, C. Brennen, G. Birkhoff - E. Zarantonello, M Gurevich, G. Logvinovich, A. Keller , S Kinnas, M. Tulin, Yu. Yakimov…

4. Asymptotic approach

Note here foundations by M. Van Dyke, J. Cole, H. Ashley and M. Landahl .. Key prevalence of asymptotic approach is that all dependencies are right for any very slender cavities what usually we have in experiments on very high speeds Asymptotic approach base is consideration cavitator and cavity as slender body and we can characterize this surface with help single slenderness parameter . But really this surface contain 2 independent enough parts where we can change independently cavitator ε (in particular for cone $\varepsilon = tg\gamma$, γ - cone semiangle) and cavity δ or $\sigma \to 0$. Asymptotic approach on base of (MAEM) V. Serebryakov (1973-91) includ 2 base cases:

1. Regular case: $\delta/\varepsilon = O(1)$, ($\sigma$=O($\varepsilon^2 \ln 1/\varepsilon$)), L=O(1) - here ε, δ and cavity sizes can not be very different.

2. Singular case: $\delta/\varepsilon \to 0$, $(\sigma << \varepsilon^2 \ln 1/\varepsilon)$ $L = O(1)$ This case is the most appropriate in case of alike disk cavitators, but it is applicable and for slender cavitator too but for considerable more slender cavities. Here for $\delta/\varepsilon \to 0$ cavitator become very small as compared to cavity length as $l = O(\delta^2 \sqrt{\ln 1/\delta})$. Solution have very complicated asymptotic structure contained 3 characteristic parts: inner near cavitator depended on cavitator form solution(for disk nonlinear solutions), interstitial solution is asymptotic at infinity (3.3), and external for middle part is perturbation of ellipsoidal cavity. Every solution for according zone is looked for separately. First it is looked for inner near cavitator solutions in zone where we have initial conditions and which therefore complete, after that this solution is matched with interstitial and it is found constants of this solution, after that complete interstitial solution is matched with external solution making it complete. Finally evenly suitable solutions contained this base parts is constructed. Note that main part of (IDE) are namely differential part. This fact define two key alternatives: from one hand it is developed second order theory on base (IDE) , from another hand it is developed very effective approach for approximation (IDE) by appropriate differential equations. The most effective here is add application of integer conservation laws, simple heuristic models and another usual for cavitation approximations.

5. Slender cavity behind slender cavitators $\delta/\varepsilon = O(1)$

Here it have been developed two approaches for given cavitation Number and for given cavity length V. Serebryakov (1973-91). Being the most simple this approach is the most frequent for application by different authors. The most universal the approach variant for different enough $r_1 = r_1(x)$, $\sigma = \sigma(x)$ (in particular for vertical cavity $\sigma(x) = \sigma_o \pm \rho g x$, g -gravity) and δ. Solutions are looked for with help rows ($x = 0$ - separation section $l = 1$)

$$\tilde{R}^2 = \tilde{R}_o^2 + \frac{1}{\ln(1/\beta^2\delta^2)}\left[\tilde{R}_1^2\right] + ... \qquad L_c = L_o + \frac{1}{\ln(1/\beta^2\delta^2)} + ... \tag{5.1}$$

transformed for the two systems of equations and in general case second order solution 2 terms is (5.4) form:

$$\frac{d^2R_o^2}{dx^2} = -\frac{2\sigma(x)}{\ln(1/\beta^2\delta^2)} \qquad \left[\frac{dR_o^2}{dx} = \frac{dr_1^2}{dx}\right]_{x=0} \qquad \left[R_o^2 = r_1^2\right]_{x=0}, \tag{5.2}$$

$$\frac{d^2R_1^2}{dx^2} = \frac{1}{2R_o^2}\left(\frac{dR_o^2}{dx}\right)^2 + \frac{d^2R_0^2}{dx^2}\ln\left(\frac{R_o^2}{4\delta^2(1+x)(1-L_o)}\right) - \int_{-1}^{0}\frac{\left.\frac{d^2r_1^2}{dx^2}\right|_{x=x_1} - \frac{d^2R_o^2}{dx^2}}{|x_1 - x|}dx_1 + \frac{\left.\frac{dR_o^2}{dx}\right|_{x=L_o}}{L_o - x}, \quad \left.\frac{dR_1^2}{dx}\right|_{x=0} = 0, \quad \left.R_1^2\right|_{x=0} = 0 \tag{5.3}$$

$$R^2 = R_0^2 + \frac{1}{\ln(1/\beta^2\delta^2)}R_1^2 = \left[\varepsilon^2 + 2n\varepsilon^2 x - \int_0^x (x - x_1)\frac{2\sigma(x_1)}{\ln(1/\beta^2\delta^2)}dx_1\right] + \frac{1}{\ln(1/\beta^2\delta^2)}\left[\int_0^x (x - x_1)\frac{d^2R_1^2}{dx^2}dx_1\right] \tag{5.4}$$

In particular for cone $r_1^2 = \varepsilon^2 (x-1)^2$, $\varepsilon = \tan\gamma$, $n = 1$ for $\sigma(x) = \text{const}$ the solutions (5.4) is complete defined by:

$$R_o^2 = \left[\varepsilon^2 + 2\varepsilon^2 x - \frac{\sigma}{\ln(1/\beta^2\delta^2)}x^2\right] = \varepsilon^2\left[1 + 2x - \sigma_\varepsilon\right] \qquad \sigma_\delta = \frac{\sigma}{\delta^2\ln(1/\beta^2\delta^2)} \qquad L_o = \frac{\sqrt{1+\sigma_\varepsilon}+1}{\sigma_\varepsilon} \qquad L_f = \frac{\sqrt{1+\sigma_\varepsilon}-1}{\sigma_\varepsilon}$$

$$L_m = \frac{2}{\sigma_\varepsilon} \qquad L_e = 2\frac{\sqrt{1+\sigma_\varepsilon}}{\sigma_\varepsilon} \qquad \frac{d^2R_1^2}{dx^2} = \varepsilon^2\sigma_\varepsilon\left\{\frac{1}{2}\left[\frac{(L_m - x)^2}{(L_f + x)(L_o - x)}\right] - 2\ln\left(\frac{\sigma_\delta}{4}\frac{(L_f + x)}{1+x)}\right) - 2\frac{1+\sigma_\varepsilon}{\sigma_\varepsilon}\ln\left(\frac{1+x}{x}\right) - \frac{L_e}{Lo - x}\right\} \tag{5.5}$$

As test it is defined analytical solutions for cone but the most effective is (5.4) with numerical calculations of the integral. Simplest case $\delta = \varepsilon$, for not slender enough cases more appropriate is $\sigma = \beta^2\delta_\bullet^2 \ln 1/\beta^2\delta_\bullet^2$. Figure 4 present solution (5.5) for cone $\alpha = 10^o, \sigma = 0.04$ - R(0,10,0.04) as compared to date of the nonlinear numerical calculations.

6. Slender cavity behind small cavitator $\delta/\varepsilon \to 0$

Outer solution describe the most part cavity and in the most part cases is enough for prediction cavity sizes and volume. Supposed cavity semi length as given $L_k = 1$ we have instead of (2.1) the statement:

$$\frac{1}{2R^2}\left(\frac{dR^2}{dx}\right)^2 + \frac{d^2R^2}{dx^2}\ln\frac{\beta^2R^2}{4(1+x)(1-x)} - -\int_{-1}^{+1}\frac{\frac{d^2R^2}{dx^2}|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1 - \frac{\frac{dR^2}{dx}|_{x=-1}}{1+x} + \frac{\frac{dR^2}{dx}|_{x=+1}}{1-x} = 2\sigma, \qquad (6.1)$$

$$\left[R^2(x)=0\right]_{x=-1},\ \left[R^2(x)=0\right]_{x=+1}$$

Solution (6.1) for $\sigma = \delta^2(\ln 1/\beta^2\delta^2)\tilde{\sigma},\ \tilde{\sigma} = O(1)$ supposed $\tilde{\sigma} = \tilde{\sigma}(x)$ is looked for as row:

$$R^2 = \delta^2\left[R_0^2 + R_{-1}^2(\ln 1/\beta^2\delta^2)^{-1} + \ldots\right] \qquad (6.2)$$

For $\tilde{\sigma} = \text{const}$ universal second order solutions applicable for different appropriate δ^2 but if $\sigma/\delta^2\ln 1/\delta^2 \to 1$):

$$R^2 = \frac{\sigma}{\ln 1/\delta^2}\left[(1-x^2) + \frac{(1-x^2) + x^2\ln 4 - \ln(1+x)^{(1-x)} \bullet \ln(1-x)^{(1-x)}}{\ln 1/\beta^2\delta^2}\right],\qquad \sigma = \frac{1}{\lambda^2}\ln\frac{1/\beta^2\delta^2}{e} \qquad (6.3)$$

First solution here was found for $\delta = 1/\lambda$ ($\beta = 1$) V. Serebryakov (1973):

$$R^2 = \frac{1}{\lambda^2}\left[(1-x^2) + \frac{x^2\ln 4 - \ln(1+x)^{(1-x)} \bullet \ln(1-x)^{(1-x)}}{\ln\lambda^2}\right],\qquad \sigma = \frac{2}{\lambda^2}\ln\frac{\lambda}{\sqrt{e}} \qquad (6.4)$$

Interstitial equations and solutions. Second order theory require of the not less 3 terms of asymptotic (3.3) (1986):

$$\frac{1}{2R^2}\left(\frac{dR^2}{dx}\right)^2 + \frac{d^2R^2}{dx^2}\ln\frac{\beta^2R^2}{4x^2} - \frac{1}{x}\frac{dR^2}{dx} = 0 \quad (6.4a) \qquad R^2 = \frac{2\sqrt{c_{do}}}{(\ln x)^{0.5}}\left(1 - \frac{\ln\ln x}{4\ln x} + \frac{\ln\frac{e\beta^2}{2}\sqrt{c_{do}}}{2\ln x} + \ldots\right) \qquad (6.4б)$$

Here we have case when asymptotic constant is defined by universal way independently on cavitator form.
Matching of (6.3-6.4б) define second order dependencies for maximal cavity radius and length) (6.5a) (1986):

$$R_k^2 = R_n^2\frac{c_d}{\sigma}\left[1 + 2\frac{\ln 2/\sqrt{e}}{\ln 1/\beta^2\delta^2}\right]\quad L_k = R_n\frac{\sqrt{c_d\ln 1/\beta^2\delta^2}}{\sigma}\left[1 - \frac{\ln e/2}{\ln 1/\beta^2\delta^2}\right]\ \ (a)\quad \lambda^2 = \frac{1}{\sigma}\ln\frac{\lambda^2\left(1 + 1/\ln\lambda^2/\beta^2\right)}{e\beta^2}\ \ (b) \qquad (6.5)$$

Asymptotically equivalent as (6.5a) dependencies are defined too by variation approach A. Petrov (1986). Process of constricting of the first order evenly suitable solution (inner + intermediate, $R_n = 1$) is demonstrated as:

$$\bar{R}^2 = \frac{2\sqrt{c_{do}}}{\sqrt{\ln\bar{x}}}\bar{x}\ \ (a) \qquad \bar{R}^2 = c\bar{x} - \frac{\sigma}{\ln 1/\sigma}\bar{x}^2\ \ (b) \qquad \bar{R}^2 = \frac{2\sqrt{c_{do}}}{\sqrt{\ln\bar{x}}}\bar{x} - \frac{\sigma}{\ln 1/\sigma}\bar{x}^2\Big|_{\bar{x}\to\infty} \to \frac{\bar{x}}{\sqrt{\ln\bar{x}}}\ \ (c) \qquad (6.6)$$

and here there are solutions for vertical cavities too. For improving of solutions for not slender enough cavities it is possible to apply $1/\delta^2 = \left[1 + 1/\left(\ln 1/\beta^2\delta^2\right)\right]$, in doing so dependence for λ (6.3) is transformed to (6.5b).

7. Dependencies for cavity sizes for alike disk cavitator

For practical calculation for alike disc cavitators the direct dependence's on σ are the most suitable. In doing so main cavity sizes λ, R_k, L_k depend on two characteristic values $\mu(\sigma)$, $k(\sigma)$ only:

$$\mu = \frac{1}{2}\ln\frac{4/e}{\beta^2\sigma}\ (a)\quad \lambda^2 = \frac{2\mu}{\sigma} = \frac{1}{\sigma}\ln\frac{4/e}{\beta^2\sigma}\ (b)\quad k = 1/\left[1 + 2\frac{\ln 2/\sqrt{e}}{\ln 5/\beta^2\sigma}\right]\ (c)\quad R_k = R_n\frac{\sqrt{c_d}}{\sqrt{k\sigma}}\ (d)\quad L_k = R_n\frac{\sqrt{c_d 2\mu/k}}{\sigma}\ (e)\ \ (7.1)$$

Figure 1, Figure 2 present calculation results for $\lambda(\sigma)$, $\mu(\sigma)$, $k(\sigma)$. Here λs correspond (6.4), λse - (6.5b), λsa, μ, ksa - (7.1) for $\beta = 1$. As tests in the paper for case cavity behind disc it is applied date of nonlinear numerical calculation by L. Guthevsky (1979): λgz, kgz, Rgz for ordinary steady cavity behind dick with Riabyshinsky scheme of closure. In doing so Figure 1 illustrate essential improving of dependence (6.5b) as compared to (6.4) for case not slender enough cavities what is important as tendentious for constricting of unsteady solutions and especially essential for R_k dependence where results jet more is improved for $\delta^2 = \sigma/\left(\ln 1/\beta^2\delta^2\right)$.

8. Equations of cavity section independent expansion.

(IDE) of (2.1, 2.2) type and accordingly (6.1) have main for $\delta \to 0$ differential part. Key idea here is some appropriate approximation of (IDE) in the out area by more simple differential equation using for getting of initial

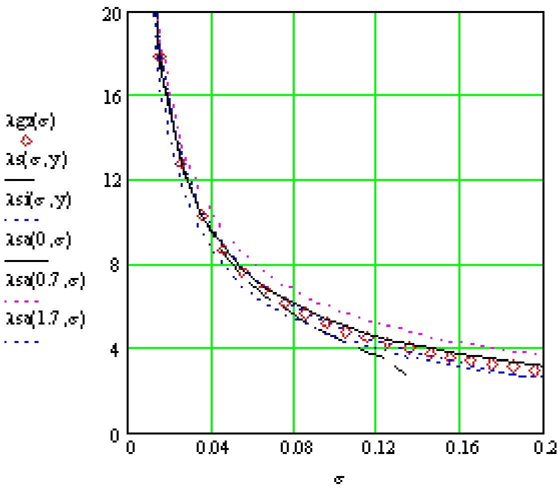

Figure 1: Dependences for aspect ratio

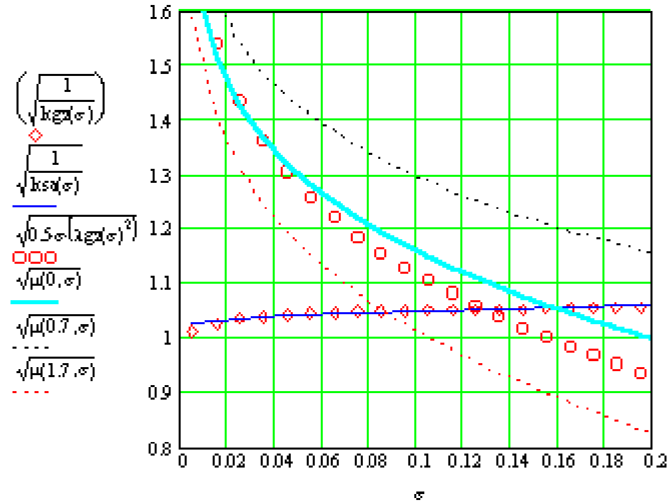

Figure 2: Slowly changing values

conditions energy conservation law. System of equations for slender cavity behind small cavitator and it's solution $\sigma = \text{const}$ defining known ellipsoidal cavity and dependence for R_k V. Serebryakov(1972-74) are:

$$\mu\frac{d^2R^2}{dx^2} + \frac{\Delta P(x)}{\rho U_\infty^2/2} = 0 \qquad R^2\Big|_{x=0} = 0, \quad \frac{dR^2}{dx}\Big|_{x=0} = 2\sqrt{\frac{D}{k\pi\mu\rho U_\infty^2}} \tag{8.1}$$

$$R^2 = R_n\sqrt{\frac{2c_d}{k\mu}}\cdot x - \frac{\sigma}{2\mu}x^2 \qquad R_k = R_n\sqrt{\frac{c_d}{k\sigma}}, \quad L_k = R_n\frac{\sqrt{2\mu c_d/k}}{\sigma}, \quad \lambda = \sqrt{\frac{2\mu}{\sigma}} \tag{8.2}$$

Here μ have clear physics as some inertial coefficient for expansion of cavity sections and at the same time as value what slowly enough is changed along changing of cavity aspect ratio. Values μ, k are defined by conditions of best from definite point of view approximation of (IDE) by differential equations and usually on base second order dependencies for cavity sizes. In particular for $\sigma = \text{const}$ the most effective are dependencies on base (7.1a) and $\mu \approx 0.5\ln\left(1.5/\beta^2\sigma\right)$.

9. Possibilities of prediction of prolate unsteady cavities

Unsteady axisymmetric cavities of alternate pressure. Practical independent of cavity sections expansion is base of very effective methods for prediction of unsteady cavities. Steady solution discover μ k as very week dependencies and in doing so prolong transportation energy in the fluid defining by $k \sim 0.95 - 1$ is small. This is demonstrate Figure 2 for values defining the main cavity sizes. Of course instead of full independence of expansion for slender cavities we have week enough dependence only mainly on cavity aspect ration but on this base it is possible to formulate differential equations for unsteady cavities too. This equations in the coordinate system connected with motionless fluid G. Logvinovich , V.Serebryakiv (1973-75) are:

$$\mu\frac{\partial^2R^2}{\partial t^2} + 2\frac{\Delta P(x,t)}{\rho} = 0\,, \qquad R^2\Big|_{t=t_n(x)} = 0, \quad \frac{\partial R^2}{\partial t}\Big|_{t=t_n(x)} = 2\sqrt{\frac{D(x)}{k\pi\mu\rho}} \tag{9.1}$$

$t = t_n(x)$ is defined as moment of passing through of some section x by cavitator having drag $D(x)$. With account of the most applicable range of $\lambda \sim 5 - 20$ here $\mu \sim 2$ frequently is applied. More accurate it is possible to apply $\mu = \mu(x)$ on base stationary dependence $\mu \approx 0.5\ln\left(1.5/\beta^2\sigma\right)$ for guasisteady values of σ. Here it have been developed more accurate ways for estimations of $\mu = \mu(x)$ for different characteristic case of flows with acceleration, vertical cavities …. Equation (9.1) on base steady dependencies for μ are applicable for very vide range of practical cases and give likelihood results even in particular for alike step changing of cavitator speed and drag coefficient . The equations are verified very good and usual accuracy here is till 5-7%. In doing so not large nonlinear part of cavity form always can be added at the final step. This equations for the first time give the possibility to predict cavities of alternate pressure and were applied for the cavity pulsation theory. *Characteristic solutions* Many of analytical solutions were obtained here for different practical cases for motion with alternate speeds and cavitator drag, for alternative pressure changing along alike step , delta, harmonic oscillations … dependencies, for vertical cavities, it was estimated influence of week explosion and shock wave on cavity and another cases. For cavities of constant on t pressure difference solution is universal as:

$$R^2 = 2\sqrt{\frac{D(x)}{k\pi\mu(x)\rho}}\left[t - t_n(x)\right] - \frac{\Delta P(x)}{\rho\mu(x)}\left[t - t_n(x)\right]^2 \qquad (9.2)$$

For the harmonic pressure oscillation ($x = 0$ at nose, normalization relay to $L_k = 1$ for cavity with σ_o) the wave

$$\sigma = \sigma_o + \sigma_s \sin \nu t, \quad R^2 = \frac{\sigma_0}{2\mu}\left[x(2 - x)\right] + \frac{\sigma_s}{\mu\nu^2}\left\{\left[\sin \nu t + \sin \nu(-t + x)\right] - \nu x \cos \nu(-t + x)\right\} \qquad (9.3)$$

start run along cavity surfaces . For the alike step changing of speed or cavitator drag cavity form have step too but have no it for alike step changing of pressure. For alike delta pressure changing cavity have concentrated alike delta perturbation directed to inside of cavity what can wet body surface and is dangerous for applications.

Not axisymmetric cavity deformation. Effective is estimation of cavity axis lifting h on base impulse conservation law G. Logvinovich (1969) , in particular of cavitators under attack angle for lateral force D_y and also gravity:

$$h_\alpha = -\frac{D_y}{\pi\rho U_\infty^2}\int_0^x \frac{dx}{R_o^2(x)} \quad (a) \qquad h_g = \frac{g}{\pi U_\infty^2}\int_0^x\left[\frac{1}{R_o^2(x)}\int_0^x R_o^2(x)dx\right]dx \quad (в) \qquad (9.4)$$

General theory of small not axisymmetric cavity perturbation as effected different factors was developed by Yu Shoravlev (1973) and was improved by V. Voronin. Note known investigations by M. Tulin, Q. Ye and Z. Cheng..

Cavities under gravity influense. Important for estimation gravity action here are dependence for minimal σ_{min} G. Logvinovich (1969) and dependence for cavity axis lifting in particular by Yu. Zguravlev

$$\sigma_{min} = \frac{2gR_k}{U_\infty^2} \quad (a) \qquad h_g = 0.33\left(x / L_k\right)^2 \frac{gL_k}{U_\infty^2} L_k \quad (b) \qquad (9.5)$$

Action of prolong gravity for penetration was considered in particular by H. Abelson,(1970), Yu. Zguravlev (1973…Cavity form (9.5c) here is defined easy enough by integral (9.2) (axis x is directed down):

$$R^2 = 2\sqrt{\frac{D(x)}{k\pi\mu(x)\rho}}\left[t - t_n(x)\right] - \frac{\Delta P_o + \rho g x}{\rho\mu(x)}\left[t - t_n(x)\right]^2 \ (a) \quad R^2 = 2\sqrt{\frac{D}{k\pi\mu\rho}}x - \frac{\Delta P_o}{2\rho\mu}x^2 \mp \frac{\rho g x}{3\rho\mu}x^3 \ (b) \quad \sigma Fr_L^2 = 4/3 \ (c) \qquad (9.5)$$

Steady vertical cavity is cavity of alternate pressure. Note here in particular investigations by A. Acosta (1961), C. Leno- R. Street (1967), O. Kiselev (1969) … Cavity form here (9.5c) is defined by equations (8.1) but here is essential influence of gravity on μ . Interesting here is possibility of negative σ at the forward part of cavity for lifting. Essential is dependence (9.5c) I. Efremov -V. Serebryakov (1978) (Fr_L - Frood Number is relay to cavity length) defining cavities with back or forward sharp ends that correspond to absent for ideal case losses of energy to the cavity wake or possibility of creating cavity for ideally case with zero cavitator. drag..

10. Cavities for $\sigma = 0$

This solutions are needed for movement in the forward part of large cavities and especially for very high speeds.

Cavities behind slender cavitator . First order solution as expansion on ε is universal and independent on cavitator form . (In particular for cone dependence (c) $\varepsilon = tg\gamma$, n=1, $x = 0$ at the nose, cone length $\ell = 1$)

$$R^2 = \varepsilon^2\left[\frac{2nx}{\sqrt{\tilde{s}}} + (1 - 2n)\right] , \qquad \tilde{s} = \frac{\ln(x/\varepsilon^2)}{\ln(1/\varepsilon^2)} ; \qquad R^2 = \varepsilon^2\left[2x\sqrt{\frac{\ln 1/\varepsilon^2}{\ln x/\varepsilon^2}} - 1\right]_{x\to\infty} \sim \frac{x}{\sqrt{\ln x}} \quad (c) \qquad (10.2)$$

Second order solutions in particular for cone $\beta = O(1)$ is:

$$R^2 = \varepsilon^2\left\{\left[\frac{2x}{\sqrt{\tilde{s}}} - 1\right] + \frac{1}{\ln 1/\beta^2\varepsilon^2}\left[\left(\frac{2x-1}{2}(\ln 2x - 1) + (x - 1)^2(\ln x - 1) - x^2 \ln x\right) + \left(\frac{\left(\ln 4/e^2\beta^2\right)x}{\sqrt{\tilde{s}}} - \frac{1}{2}\frac{x\ln\tilde{s}}{\tilde{s}\sqrt{\tilde{s}}} + \frac{x\ln\beta^2 e/2}{\tilde{s}\sqrt{\tilde{s}}}\right) - \left(x\ln\frac{2}{e} - x\ln x\right)\right]\right\} \quad (10.3)$$

Cavities behind alike disc cavitator. For inner nonlinear near alike disk cavitator solution semi heuristic approach based on flow near paraboloid is applied with help PLG transformation. Equation to be obtained here is (10.4a)

$$\frac{dR^2}{dx}=\frac{2\sqrt{c_{do}}R_n}{\sqrt{\ln 4(x+\Delta)^2/\beta^2R^2}}\to\frac{2R_n\sqrt{c_{do}}}{(\ln x)^{0.5}},\quad \Delta=0.5R_n\left[\sqrt{c_{do}}+1/\sqrt{c_{do}}\right],\quad R^2\big|_{x=0}=R_n^2;\ \text{(a)}\quad R_o^2=1+2\sqrt{c_{do}}\,x\Bigg/\left(\sqrt{\ln\frac{4(x+\Delta)^2}{\beta^2\left(1+2\sqrt{c_{do}}\,x\right)}}\right)\ \text{(b)}\qquad(10.4)$$

and the first order solution this equations is (10.4b) where all values are normalized relay to R_n at the separation section.. This dependencies are applicable for different alike disk cavitator and for not fixed separation section too. Figure 5. illustrate accuracy of calculation on base equations (10.4a) - Rd as compared to date of nonlinear numerical calculation - Rg ; Rb,Rl experiment date by C. Brennen (1969), G. Logvinovich (1969) .

11. Effective calculation of cavities behind alike disk cavitator

Nonlinear numerical calculation of supercavitation occupy especial place. There are here two essential things. First taking into account very small gap between body and cavity there are necessity of very accurate calculation what here is hard enough problem From another hand fluid is real fluid with viscosity, capillary …. and in doing so ideal model predict infinite curvature at separation section where viscous and capillary effect can be essential . Nature closure with chaotic at the cavity end flow and essential losses of pressure is very different as compared to ideal.. Nevertheless both of accurate enough nonlinear numerical solution and accounting the effects out of this not enough perfect ideal model are important enough. First note here investigations by C. Brennen (1969), L. Guthevsky (1979). At present the most part of calculation methods was developed mainly for steady flows and here it was accumulated essential experience: E. Amromin - A. Ivanov , V. Bushkovsky, E. Block L. Cozguro, Yu. Deinekyn, R. Jeppson, V. Shepelenko, A.Terentiev- N. Dimitrieva, G. Todorashko, Yu. Zuikov … Alike methods have double difficulty so for solution of definite problem here it is need at the same time to overcome and hard enough problem of needed accuracy and this is especially essential for considerably more many variants unsteady flows. So here it is developed linearized approach based on more appropriate approximation of (IDE) where the integral part is defined on base outer expansions. For general enough case of unsteady flows system (9.1) can be improved as:

$$\frac{\partial}{\partial t}\left[\sqrt{\mu(x,t)}\frac{\partial R^2}{\partial t}\right]+\frac{1}{\sqrt{\mu(x,t)}}\frac{2\Delta P(x,t)}{\rho}=0,\quad \left.\frac{\partial R^2}{\partial t}\right|_{t=t_n(x)}=2\sqrt{\frac{D(x)}{k(x)\rho\mu_n}},\quad R^2\big|_{t=t_n(x)}=R_n^2,\ \text{(a)}\quad R^2\big|_{t=t_c(x)}=R_n^2\ \text{(b)}$$

$$\mu(x,t)=\frac{1}{2}\ln\frac{4\left[(x_n+\Delta)-x\right]^2\left[x-(x_c+\Delta)\right]^2}{\beta^2R^2\left[x_n-x_c+2\Delta\right]^2}\left[1-\frac{0.4[x_n-x][x-x_c]}{[x_n-x_c]^2}\right],\quad \mu_n=\ln\frac{2\Delta}{\beta R_n},\quad \Delta=\frac{R_n}{2}\left(\sqrt{c_d}-\frac{1}{\sqrt{c_d}}\right)\qquad(11.1)$$

Here solution for unsteady cavity is looked for on base simple differential equation but for add condition of alike (11.1b) type. This system give the possibility to predict wide enough range of cases but require further perfecting for cavities of alternate pressure and especially condition (11.1b). k here can be accounted by quasistationary way.

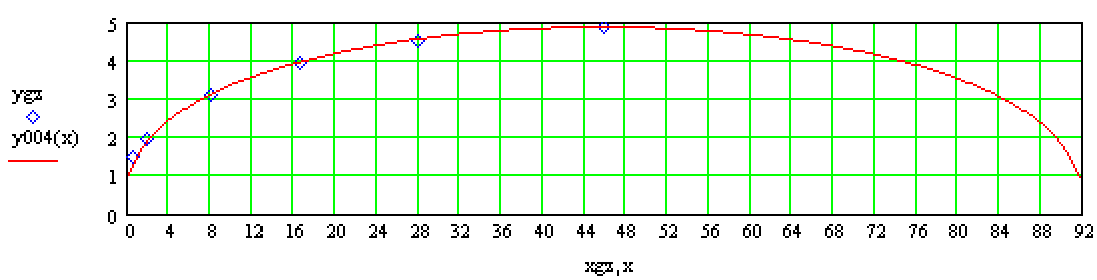

Figure 3: Cavity behind disk prediction

Figure 3: solution of the system (11.1) for $\sigma = 0.04$ - $R = y004(x)$, nonlinear numerical calculation $R = ygz$.

12. Unsteady cavities with gas injection

Gas injections give the possibilities considerably to increase the cavity sizes and accordingly effectiveness and range of supercavitation application. The most considerable influence here have gas losses processes from back part of cavity what the most strongly depend on gravity influence considerably limiting real possibility of gas injection. Note here papers by M. Billlet, C. Brennen, L. Epstain, G. Logvinovich, J.M. Michel, J.P. Franc, J. B. Paquet - J.P. Flodrops - A. Dyment, Yu Savchenko,D. Stinebring . For changing of the speed , outer pressure, gas injection - cavity sizes and form are changed. This changing for definite conditions is accompanied by oscillations of pressure in cavity and creating of waves what start run on cavity surfaces. Note here known investigations by J.M. Michel, E.Silberman -C.Song, L. Woods andcavity pulsation theory by Paryshev (1978). Note here papers by D.Dianov, V.Semenenko too. Key moment for unsteady cavity with gas injection prediction is problem of defining of pressure in cavity $P_c(t)$. Polytropic dependence for gas in cavity is effective enough. In case for simpli-ciity of cavity without body- statement of the problem of the cavity prediction (defining P_c) for given movement , outer pressure , gas injection contain equations (9.1) and equation of conservation of gas mass in cavity:

$$R^2\Big|_{x=x_c(t)}=0,\;\; u=\int_{x_n(t)}^{x_c(t)}\pi R^2 dx\;\;,\;\; \frac{d}{dt}\Big(\frac{uP_c}{\alpha_t}\Big)=(Q_{m\_in}-Q_{m\_out}),\;\; \frac{uP_c}{\alpha_t}\Big|_{t=0}=m_{c0} \qquad (12.1)$$

where $x_n(t), x_c(t)$ laws of cavitator and cavity end motion; $P_c=\alpha_t\rho_c$ isothermal dependence pressure and mass density ρ_c of gas in the cavity; $uP_c/\alpha_t=m_c(t)$, m_c, u - mass, volume of gas in cavity Q_{m_in}, Q_{m_out} - mass gas injection and losses at the cavity end. Besides this equation it is need add by dependence for mass gas lossess from the cavity. With account of definite independence of the back part of cavity and used G. Logvinovich formula more improved dependence for gas loses connected namely with parameters of back part of cavity is found

$$Q_{m\_out}\approx k_\xi\rho_e R_n^2 U_0\frac{(1-\sigma/E)}{\sigma}\frac{\rho}{P_\infty-P_c}\left(\frac{\partial R^2}{\partial t}\right)^2\Bigg|_{x=x_c(t)} \qquad (12.2)$$

$k_\xi\approx\text{const}$ defined on base stationary experiment, , ρ_e - mass density for the outer hydrostatic pressure near cavity end , E - Euler Number. Solutions of equations (9.1,12.1.12.2) as whole define $P_c(t)$ and after that cavity is defined on base equation (9.1) or more accurate: system of (11.1). The most simple is to define solutions for quasistationary statement on base simple differential equations relay to $\sigma=\sigma(x_n(t))$ for given laws of gas injection:

$$Q_{m\_in}=Q_{m\_in}(x_n(t))\,,\quad \bar{Q}_{m\_in}=Q_{m\_in}/U_0R_n^2\rho_{ae}\quad (a)\quad \frac{d}{d\bar{x}_n}\left(\frac{1-\sigma/E}{\sigma}\right)+0.15k_s\left(\frac{1-\sigma/E}{\sigma}\right)=0.15\bar{Q}_{m\_in} \qquad (12.3)$$

$\bar{x}_n=x_n/R_n$ k_s - experimental constant. Physics (12.3)first the quick changing of the cavity is appeared under action of unsteady gas injection and gas losses from the cavity end . When gas injection become as constant with new intensity - considerably more slowly evolution of the cavity directed to new stable position is started. Key point for prediction of unsteady cavity with gas injection is gas losses prediction problem very connected with singularities of flow at cavity closure and vortex-creating processes. Note here papers by Arndt R.E.A. , Arakery V.H.and Higushi H. (1991) , Callenaere M., Franc J.P., Michel J.M (1998) Laberteaux, K.R., Ceccio S.L (1998).

13. Base equations of compressible approach

Statement of the problem on base model of ideal izentropic compressible fluid is alike as for incompressible fluid. Key equations here accounting water specific are Tet adiabatic curve (13.1a) and compressible Bernoulli equation:

$$\frac{P+B}{\rho^n}=\frac{P_\infty+B}{\rho_\infty^n}\quad (a) \qquad\qquad \frac{n}{n-1}\frac{P+B}{\rho}+\frac{U^2+v^2}{2}=\frac{n}{n-1}\frac{P_\infty+B}{\rho_\infty}+\frac{U_\infty^2}{2}\quad (b) \qquad (13.1)$$

$B=3045\ \text{kg}/\text{cm}^2$, $n=7{,}15$. For small perturbed flows (slender bodies) for $M<1$ и $M>1$ The statement is simplified and instead of Laplace equation we have Prandtl-Glauert (13.2a) equation and in range of $M\sim 1$ transonic Karman-Guderley equation (13.b). Equations (13.2a) both for water and air are the same as acoustic equations.

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\left(1\text{-}M^2\right)\frac{\partial^2\varphi}{\partial x^2}=0\quad (a) \qquad \frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\left[\left(1-M_\infty^2\right)-\frac{(n+1)M_\infty^2}{U_\infty}\frac{\partial\varphi}{\partial x}\right]\frac{\partial^2\varphi}{\partial x^2}=0\quad (b) \qquad (13.2)$$

$$\frac{1}{4R^2}\left(\frac{dR^2}{dx}\right)^2+\frac{1}{2}\frac{d^2R^2}{dx^2}\ln\frac{B^2R^2}{4x^2}-\int_0^{x_0}\frac{\frac{d^2r_1^2}{dx^2}\big|_{x=x_1}-\frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1-\;-\int_{x_0}^{x}\frac{\frac{d^2R^2}{dx^2}\big|_{x=x_1}-\frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1-\frac{\frac{dr_1^2}{dx}\big|_{x=0}}{x}=\sigma$$

$(\ln 1/\delta)^{-1}$ $\quad(1)\quad$ $(\ln 1/\delta)^{-1}$ $\quad$ $(\ln 1/\delta)^{-1}$ $\quad$ $(\ln 1/\delta)^{-1}$ (1) $\qquad$ (13..3)

Equation (13.2a) on base (SBT) is transformed to IDE : for $M<1$ this is (2.1); for $M>1$ $B^2=|M^2-1|$ is (13b) for analogous as (2.1) initial conditions at separation section and condition at closure.

14. Sub- and supersonic supercavitation flows

As follow (2.1,13.3) asymptotic structure of (IDE) for $M<1$ $M>1$ are analogous as for $M=0$. This fact give the possibility to apply all technology developed for $M=0$ and develop complete enough second order approach V. Serebryakov (1990-94). *Interstitial equations and solutions* discover can be considerable narrow for $M>1$ forward parts of cavities for $\sigma=0$ as compared to $M<1$ what can be explained appearing of wave losses:

$$M>1 \quad \frac{1}{4}\frac{1}{R^2}\left(\frac{dR^2}{dx}\right)^2+\frac{1}{2}\frac{d^2R^2}{dx^2}\ln\frac{B^2R^2}{4x^2}-\frac{1}{x}\frac{dR^2}{dx}=0 \text{ (a)}, \quad R^2=\frac{K_S}{(\ln x)^{1.5}}\left[1-\frac{9}{4}\frac{\ln\ln x}{\ln x}+\frac{3}{2}\frac{\ln K_S B^2/4}{\ln x}\right]\sim\frac{x}{(\ln x)^{1,5}} \text{ (b)} \quad (14.2)$$

Alike as for $M<1$ solutions to the limit and second order solution in case of cone for $M>1$ $\sigma=0$ are found:

$$R^2=\varepsilon^2\left[\frac{2\,nx}{(\tilde{s})^{1.5}}+(1-2n)\right] \qquad \tilde{s}=\frac{\ln(x/\varepsilon^2)}{\ln(1/\varepsilon^2)} \qquad R^2=\varepsilon^2\left[2nx\left(\frac{\ln(1/\varepsilon^2)}{\ln(x/\varepsilon^2)}\right)^{1.5}+(1-2n)\right]_{x\to\infty}\sim\frac{x}{(\ln x)^{\mathbf{1.5}}} \quad (14.\ 3)$$

$$R^2=\varepsilon^2\left\{\left[\frac{2x}{\tilde{s}\sqrt{\tilde{s}}}-1\right]+\frac{1}{\ln(B^2/\varepsilon^2)}\left[\left((x-1)+\frac{2x-1}{2}(\ln 2x-1)+2(x-1)^2(\ln x-1)-2x^2\ln x\right)+\left(\frac{\left(3\ln 2/B^2+\ln 2/e\right)x}{\tilde{s}\sqrt{\tilde{s}}}-\frac{9}{2}\frac{x\ln\tilde{s}}{\tilde{s}^2\sqrt{\tilde{s}}}+3\frac{x\ln B^2/2}{\tilde{s}^2\sqrt{\tilde{s}}}\right)-\left(x\ln\frac{2}{e}-3x\ln x\right)\right]\right\} \quad (14.4)$$

Second order solution (14.4) define for $x\to\infty$ value of K_s for cone case:

$$K_S=2\varepsilon\left[\ln\left(\frac{2}{e}\right)^{1/3}\frac{2}{B^2\varepsilon^2}\right]^{1.5}\sim 2\varepsilon\left(\ln\frac{1}{B^2\varepsilon^2}\right)^{1.5} \qquad K_s=2\sqrt{c_{do}}\ln\frac{2}{B^2\varepsilon^2} \quad (14.5$$

Figure 5 Illustrate calculation of dependence (14.4) for $M=2$ - Rs(z,0.176) cone with $\gamma=10^o$, Ra - nonlinear numerical calculation by G. Aleve (1983), Ri(x,0.176) first term of solution (10.3) for $M=0$. Calculation show not essential influence of compressibility on forward part of cavity for $M<1$. But for $M>1$ this influence can made forward part of cavity for $M>1$ considerably more narrow as compared to case of $M<1$.

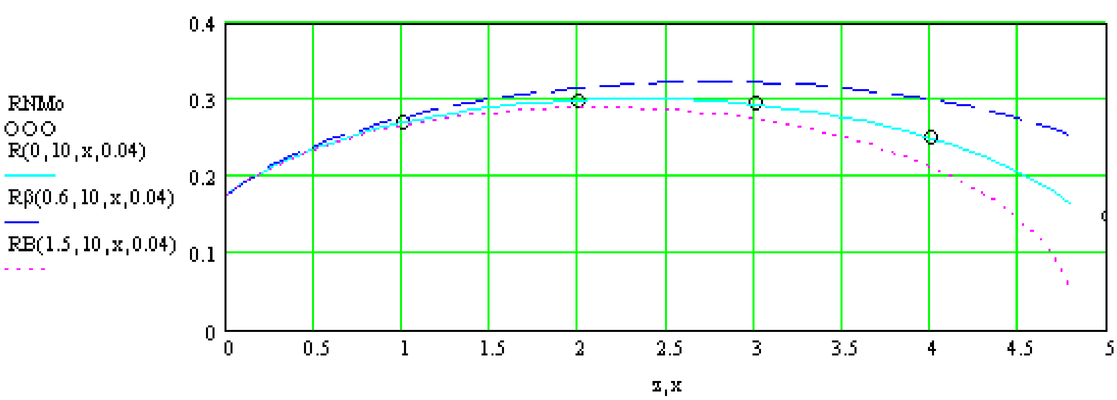

Figure 4: Slender cavity behind slender cone prediction

Figure 4 : Illustrate compressibility effects for $M<1$ $M>1$ for cone $\alpha=10^o$, $\sigma=0.04$ in case of finite cavity solution (5.5): $M=0.6$ - Rβ(0.6,10,x,0.04) ; solution (14.7) $M=1.5$ - RB(1.5,10,x,0.04) as compared to case of $M=0$ Influence of compressibility for $M<1$ can essentially increase sizes of cavities and essentially decrease - for $M>1$.

For slender cavity and cavitator solution for $M>1$ repeat (5.4,5.5) being different by second order terms only:

$$M>1 \qquad \frac{d^2R_1^2}{dx^2}=\frac{1}{2R_o^2}\left(\frac{dR_o^2}{dx}\right)^2+\frac{d^2R_0^2}{dx^2}\ln\left(\frac{R_o^2}{4\delta^2(1+x)^2}\right)-2\int_{-1}^{0}\frac{\left.\frac{d^2r_1^2}{dx^2}\right|_{x=x_1}-\frac{d^2R_o^2}{dx^2}}{|x_1-x|}dx_1 \quad (14.6)$$

and in case of cone is:

$$M>1 \qquad \frac{d^2R_1^2}{dx^2}=\varepsilon^2\sigma_\varepsilon\left\{\frac{1}{2}\left[\frac{(L_m-x)^2}{(L_f+x)(L_o-x)}\right]-2\ln\left(\frac{\sigma_\delta}{4}\frac{(L_f+x)(L_0-x)}{(1+x)^2}\right)-4\frac{1+\sigma_\varepsilon}{\sigma_\varepsilon}\ln\left(\frac{1+x}{x}\right)\right\} \qquad (14.7)$$

Slender cavity behind small cavitator Outer solution is defined by analogous as (6.1, 6.3) way:

$$R^2=\frac{\sigma}{\ln 1/\delta^2}\left[(1-x^2)+\frac{(1-x^2)+x^2\ln 4+\ln(1+x)^{(x^2-x-2)}\bullet\ln(1-x)^{(x-x^2)}}{\ln 1/B^2\delta^2}\right] \quad (a) \qquad \sigma=\frac{1}{\lambda^2}\ln\frac{1/\delta^2}{e\,|M^2-1|} \quad (b) \qquad (14.8)$$

Where aspect ratio is defined by the same $M<1$ $M>1$ dependence (14.8b). I particular case $\delta=\varepsilon$ outer solution after the manner (6.4) was obtained by A. Vasin (1987) too. Dependencies on base of matching of the second order solutions (14.2) and (14.4) and in particular for cone (14. 8) define cavity maximal radius for $M>1$:

$$\frac{R_k}{R_n}=\frac{1}{\sqrt{\sigma}}\frac{K_s/2}{\ln 2e/B^2\delta^2} \ (a) \quad \frac{R_k}{R_n}=\sqrt{\frac{c_{do}}{\sigma}}\frac{\ln 2/B^2\varepsilon^2}{\ln 2e/B^2\delta^2} \ (b) \quad \frac{R_k}{R_n}=\sqrt{\frac{c_{do}}{k_B\sigma}} \quad k_B\sim\left(\frac{\ln/B^2\delta^2}{\ln/B^2\varepsilon^2}\right)^2, \quad k_B|_{B\to 0}\to 1, \quad k_B|_{\substack{\sigma\to 0,\\ \delta\to 0}}\to\infty \ (c) \quad (14.8)$$

Alike as (6.6) it is constructed asymptotic structure of evenly suitable solution (normalization relay to R_n)

$$\bar{R}^2=\frac{K_S}{(\ln\bar{x})^{1.5}}\bar{x} \ (a), \qquad \bar{R}^2=c\bar{x}-\frac{\sigma}{\ln 1/\sigma}\bar{x}^2 \ (b), \qquad \bar{R}^2=\frac{K_s}{(\ln\bar{x})^{1.5}}\bar{x}-\frac{\sigma}{\ln 1/\sigma}\bar{x}^2\Big|_{\bar{x}\to\infty}\sim\frac{1}{(\ln\bar{x})^{1.5}}\bar{x} \ (c) \quad (14.9)$$

Alike as (6.8) for $M>1$ it is possible to suppose good enough results for estimation by next dependencies:

$$\mu=\frac{1}{2}\ln\frac{4/e}{|M^2-1|\sigma} \ (a), \ \lambda^2=\frac{2\mu}{\sigma}=\frac{1}{\sigma}\ln\frac{4/e}{|M^2-1|\sigma} \ (b), \ k_B=\left(\frac{\ln 2e/B^2\delta^2}{\ln 2/B^2\varepsilon^2}\right)^2 (c), \ R_k=R_n\frac{\sqrt{c_d}}{\sqrt{k_B\sigma}} \ (d), \ \delta^2=\frac{\sigma}{(\ln 1/B^2\delta^2)} \ (e) \quad (14.10)$$

where however dependence (14.10c) is on the base of case cone solution.

15. Prediction possibilities of the supercavitation with account of compressibility

First investigations of compressibility effects in water were connected with explosion in and penetration into water problems. For strike against water of blunted bodies even for very moderate speed the stresses in several time more as compared to steady flow and can reach enormous values. There are considerable experience in this field : G. Aleve C. Chou, V. Eroshin ,O. Faltinsen, , E Fontaine - R. Cointe, V. Kubenko , A. Korobkin, H. Pfeifer - M. Shaffar, V. Poruchikov, A. Sagomonian , O. Shorugin, , Yu Yakimov .

Properties of very high speed water flow are strongly different as compared to air. Water is considerable more rigid, inner energy very small, considerable part of flow can be near as for incompressible case, strong dependence sonic speed depend on pressure and large distances till normal shock wave, small energy losses in shock wave - adiabatic and shock adiabatic curves practically the same. .For motion with very high speed enormous *hydrodynamic stresses* are appeared even for steady flow but in not large zones near alike disk cavitator where nonlinear effects are concentrated. For whole another area flow we have small mainly acoustic flow perturbations.

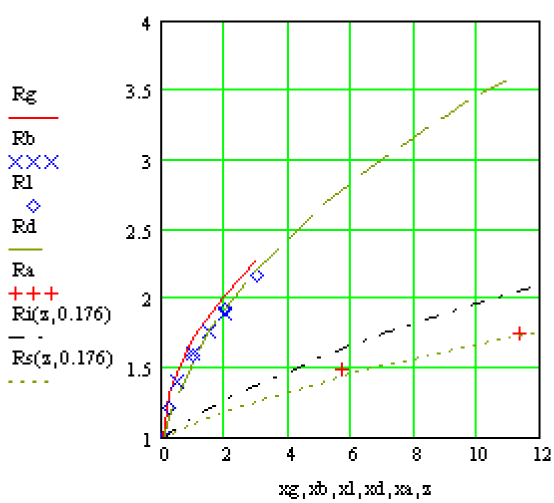

Figure 5: Forward part of cavities

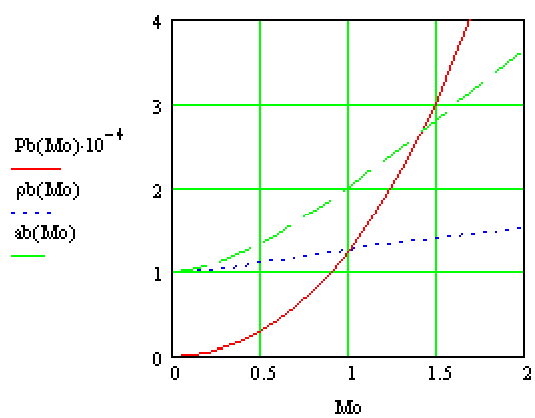

Figure 6: Braking point values

Main peculiarities of flows in nonlinear zones are demonstrated by dependencies for pressure, mass density, sonic speed in the break point P_*, ρ_*, a_*, illustrated by Figure 6: $P_* = Pb$, $\rho_*/\rho_\infty = \rho b$, $a_*/a_\infty = ab$, ∞ - indicate at infinity:

$$P_* = \frac{\rho_\infty U_\infty^2}{n M_\infty^2}\left[\left(1+\frac{n-1}{2}M_\infty^2\right)^{\frac{n}{n-1}} - 1\right] \quad (a) \qquad \frac{\rho_*}{\rho_\infty} = \left(1+\frac{n-1}{2}M_\infty^2\right)^{\frac{1}{n-1}} \quad (b) \qquad a^2 = a_\infty^2\left(1+\frac{n-1}{2}M_\infty^2\right) \quad (c) \quad (15.1)$$

Even acoustic approach only discover main *considerable differences* of compressible supercavitation flows. As follow from (6.8, 14.8, 14.10) for $M<1$ k values slightly different as compared to 1 what mean that practically all energy of cavitator drag is transformed to the potential energy at middle section of the cavity. As distinguished from $M<1$ values of k_B for $M>1$ for decreasing of σ tend to be strongly increased indicating tendition that for this case the most part of energy of cavitator drag have tendentious to go out to the wave losses. So for $M>1$ key property of supercavitation when cavity form and sizes practically depend on cavitaor drag can be considerably infringed and here it is need to account of cavitator form too. Essntial point here as follow from (14.2), Fig.5 is fact that wave losses are concentrated namely on forward part of cavities. Subsonic effect influence is not essential for forward part of cavities but is essential for finite cavitiy especially behind slender cavitator increasing it's sizes.

The transonic range $M \sim 1$ is both the most important, interesting , difficult. Thank to large adiabatic coefficient $n \sim 7.15$ this range considerable more wide as compared to air, on base G. Aleve calcullation for cone drag this is it would rather range $M \sim 0.7, 0.8 - 1.4, 1.5$. Along this range flow slowly enough is transformed from pure subsonic till mainly supersonic. Start from $M \sim 0.7, 0.8 - 1.4, 1.5$ for $\sigma \sim 0.03 - 0.02$ finite supersonic zone and accordingly shock wave at the cavity back are appeared, in doing so initial critical surfaces is cavity. For $M>1$ we have 2 finite forward and back subsonic zones and accordingly 2 normal shock waves. There are for transonic very wide lateral sizes of the flow. As follow from (6.8, 14.8, 14.10) for both cases $M<1$ $M>1$ values of k tend to 1 indicated decreasing in range of $M \sim 1$ both prolong transportation energy and wave losses. At the same time value aspect ratios on base of (SBT) tend to be unlimited in range of $M \sim 1$ requiring consideration on base more accurate for this zone transonic equation (13.2b). *Experiments possibilities for very high speeds.* There are here date: Mc. Millen J.H. and Harwey E.N. (1946), Yu Yakimov, V. Eroshin (1980), Bivin Yu, Gluchov Yu, Permiakov Yu/ (1985), Savchenko Yu., Semenenko V., Serebryakov(1993), Kirschner I. (1998). All experiments for nature pressure only - very long and very slender like needs cavities and thank to it considerable influence of walls and especially for transonic. Alike disk cavitator can no be not considerably deformed for alike enormous stresses under penetration. It would rather single possibility to verify theory is forward part of experiment cavity mainly behind alike slender cone cavitator only. *Prediction methods.* Note here first results by T. Nishiyama T.- O Khan (1981), G. Aleve (1980-90) - nonlinear numerical calculation of penatration and steady flow for $M<1$, $M>1$ mainly forward cavity part, L. Zigangareeva O. Kiselev (1994-98)- nonlinear prediction of finite cavity $M<1$, A. Terentiev- A. Chechnev (1989) - nonlinear numerical prediction $M<1$, $M>1$ of penetration and finite cavity, V. Serebryakov (1989-94) - linearized approach on base (SBT) for $M<1$, $M>1$, finite and infinite cavities, A. Vasin (1996-97)- nonlinear numerical prediction in case disk cavitators, finite cavity till $M<1.2$, A. Wargnese, J. Uhlman, I. Kirschner (1997) - subsonic, finite cavity, slender cavitator. For estimation both $M<1$ $M>1$ the most simple way is equations (8.1):

$$\mu\frac{d^2R^2}{dx^2} + \frac{\Delta P(x)}{\rho U_\infty^2/2} = 0 \qquad R^2\Big|_{x=0} = 0, \; \frac{dR^2}{dx}\Big|_{x=0} = 2\sqrt{\frac{D}{k\pi\mu\rho U_\infty^2}} \qquad (15.2)$$

Alike disk cavitator drag is good estimated on base pressure in breaking point. Cavity form with good accuracy and especially for $M \sim 1$ is ellipsoid. k values alike as (14.10c) can be estimated on base acoustics approach and this values are alike to nonlinear numerical calculations date. Main point is prediction μ for $M \sim 1$. Influence of M on λ, μ for $M<1$, $M>1$ show Figure 1, 2 : $\lambda sa(0.7,\sigma)$, $\lambda sa(1.7,\sigma)$; $\mu(0.7,\sigma)$, $\mu(1.7,\sigma)$ accordingly for $M \sim 0.7, 1.7$

Estimations of μ , (λ) on base transonic equation (13.2b) by known in aerodynamic simple approach by J. Spreiter- A. Alksne (1959 are in contradiction for $M>1$ with the date of nonlinear numerical calculations till $M<1.2$ which predict tendention of considerable increasing of λ along increasing of M. The receiving of any experimental date what could verify theory for finite cavity for $M>1$ now become especially important and immediate.

16. Linearized equations of supercavitating bodies movement.

For high speed motion in water lateral hydrodynamic forces and bended stresses in prolate bodies can be considerable even for moderate speeds. Practically real is movement along sloping enough trajectories only that is essential simplification factors for constructing effective enough theory of supercavitation bodies movement on base approaches of the trajectories of small curvature . The most simple here is case of near to straight trajectories and here is possible essentially separate consideration of prolong and lateral motion.

Prolong motion Apply to launching problem in water the simplest model of motion by inertia under action of cavitator drag only (the drag is proportional to square of cavitator speed) is applied. Maximal range X_* of cavitational part of the trajectory finished for subsonic speeds is defined by dependence:

$$X_* = 0.23\left(\frac{G}{\rho_b g}\right)^{1/3} \frac{\rho_b}{\rho}(\kappa)^{2/3}\left(\ln\frac{0.71}{\sigma_0}\right)^{1/3} / \sigma_0^{4/3} , \qquad \sigma_* = \sigma_0 e^{3/4} \quad \text{(a)} \qquad (16.1)$$

G , ρ_b -weight, mass density of body, g gravity, σ_o, σ_* correspond initial and final moment where cavity and body coincide (κ - indicate filled by mass part of end cavity). Maximal values of length is reached for condition (18.1a).

Lateral motion It is important here problems of interaction of back part of body or his stabilizer surfaces with cavity. Important here is planning theory apply to motion in cavity. In incompressible fluid apply to motion in the cavity this theory was developed by E. Paryshev (1973) as theory of planing cylinder on cylindrical free surface. Note here investigations by M. Tulin, W. Vorus (1996), S. Putilin. Essential here is possibility to describe for some practical cases lateral force with help of add mass coefficient in separation streamlines section. Alike simplest theory is very effective in aerodynamics and can be applied and in supercavitation in case of wetted enough back part of supercavitating bodies for cavitation Recently the planning theory for plates of finite lengthening for sub and supersonic speed have been developed too by A. Maiboroda (2000). The theory is developed for plain trajectory of small curvature $y = y(x)$. For near straight trajectory of body with wetted back part equations are:

$$\frac{d^2\Delta\bar{y}}{d\bar{x}^2} + k_h\bar{m}\frac{d\Delta\bar{y}}{d\bar{x}} + k_h k_\Delta \frac{\bar{m}}{\bar{I}}\Delta\bar{y} = 0, \qquad \Delta\bar{y}\big|_{\bar{x}=0} = -k_\Delta \bar{I}(\alpha_0 - k_\Delta \bar{I}\bar{\omega}_0) \qquad \Delta\bar{y} = \bar{y} - \bar{I}(\alpha_0 - \bar{I}\bar{\omega}_0) - (\theta_0 + \bar{I}\bar{\omega}_0)\bar{x}.$$

$$\bar{m} = \frac{ma}{M},\ \bar{I} = \frac{I}{Ma^2},\ \bar{\omega} = \frac{\omega_0 a}{V_0}, \qquad k_h = 1/[1 + \frac{mV^2}{K_h}], \qquad \frac{1}{K_h} = [\frac{1}{K_s} + \frac{1}{K_{b\Delta}}] \qquad k_\Delta = (1 - \Delta / a) \qquad K_{b\Delta} = \xi_{b\Delta}\frac{Ei_m}{a^2} \qquad (16.2)$$

here $\alpha_0, \theta_0, V_0, \omega_0, a, m, M, I$ initial attack and trajectory angles, initial a prolong and angular speeds, distance between mass center and pressure center and its shift Δ from body base to the forward , separation add mass of back wetted part of body or stabilizers interacted with the cavity, body mass and lateral moment of inertia. E, i_m rigidity modulus, moment inertia body middle section $K_{b\Delta}, K_s$ - rigidity of stabilizer and body for motion defined by special way. In particular for case of steel cylinder with lateral force in the end section ($\Delta \sim 0$) ξ_b~4,8. Equation (15.2) define lateral coordinates of the trajectory relay to axis inclined from direction of motion at the initial moment for definite angle and is classical equation of harmonic oscillations with damping at the same time for $\alpha, \theta, (\alpha + \theta)$. and hve clear physics corresponding lateral and angular oscillations of body with energy which is trasformed from one type inti anither and go out to the energy of lateral motion of wake behind body.

17. Conclusions

For moderate speed at present base not enough investigated and immediate is problem of prediction of gas losses from unsteady cavities . For range of very high speed the most immediate and not enough solved is problem of hopeful enough prediction of supercavitation for wide enough range of $M > 1$ and especially $M \sim 1$ where the most immediate is not any solved till now problem of any experimental verification of the theory for $M > 1$ in case of finite cavities. Existing supercavitation theory both for nonlinear numerical methods and linearized approaches are essentially advanced and for near future it is real possibility to predict as whole flow and motion of high speed supercavitating bodies with help simple and accurate enough methods for whole range of the most interesting both moderate and very high Mach Numbers M~0-2.5 . The cavitational drag and drag reduction problems are out of present consideration.

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